Deductive Metaphysics Using Natural Language Logic

by Kyle Foley

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(Draft)

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1. Introduction

1.1. What We're Trying to Accomplish

The goal of this book is to turn metaphysics into a testable discipline. That is to say, we want to put an end to the days where a metaphysician can simply make an assertion, dress it up into some fancy language and because there is no clear test they must pass they will continue to persist in their error. What we propose instead is a future where the metaphysician, just like the physicist, will have to predict that given one state of affairs in nature another state will inevitably follow. In short, make predictions where success or failure is obvious.

We will use mathematics as our guide. Perhaps earlier, but at least as far back as Babbage mathematicians were making predictions. They would ask for the answer to a complex logarithm and if two people independently arrived at the same result then that added to the certainty of the mathematicians' claims. In the 1930s mathematicians acquired even more credibility when they demonstrated that they could build machines which would output expected answers.

That is how I suggest metaphysics will become a testable discipline: the metaphysician will have to build a machine which will reliably output the expected answer to easy questions using certain metaphysical concepts. When the metaphysician can reliably predict the output for the easy claims, they will thus become more credible when they assert a truth-value for a controversial claim. We have now built software which accomplishes this for 190 trivial claims which use stereotypical metaphysical concepts.

1.2. The Scope of the Following Manuscript

Although our goals are quite ambitions for the moment we are just worried about getting our logical language up and running which is why we ended the previous section abruptly. It would be inappropriate to sound off on grand metaphysical questions when we do not now have the hypotheses for their solution.

In the following 140 pages we explain the basics of our logical language which we call Natural Language Logic (NLL). Although we have arguments concerning certain metaphysical claims, there is no point right now in trying to persuade anyone of them,
simply because 190 arguments is not enough to persuade a hard-nosed skeptic of anything. What we do have, however, is a highly expressive language which has working hypotheses for how a large number of metaphysical concepts can be formalized. At present, the logic in this book covers instantiation, the indefinable terms, the determiners and a large set of irregular definitions.

1.3. The Intended Audience

The book is really only intended for potential collaborators. We are still too early in our project to demand other metaphysicians do things our way. Although I have tried to make the logic in this text as clear and as easy to understand as possible, it is unlikely that it will be understandable without exercises.

1.4. The Terminal/Decideable Distinction

We are not going to use the word 'decideable' because some authors, Hilbert chief among them, use the term to mean: 'b is decideable iff every consequence of b can be calculated in a finite number of steps and every consequence of b is true in reality'. Other authors dispense with the 'true in reality' part and are more worried about 'truth in a structure' which means something like 'b is decideable iff every consequence of b can be calculated in a finite number of steps and the consequences are only applicable to the working structure and say nothing about the external world.' We are going to use the terms 'terminal' and 'infallible' as follows:

1. b is terminal iff every consequence of b can be calculated in a short number of finite steps even if some of those consequences are counterintuitive.

2. b is infallible iff every consequence of b can be calculated in a short number of finite steps and either all or practically all consequences of b are true in reality.

We say short number of finite number of steps because it is unacceptable if a machine takes longer than roughly 10 minutes to calculate the answer, indeed even longer than 10 seconds is enormously frustrating. A system only becomes persuasive when it can calculate a large number of intuitively true consequences and if it takes 10 minutes to calculate the answer then it is unlikely that such a machine will ever be able to calculate a large number of the common sense claims that humans are interested in since building a logically perfect language requires an enormous amount of testing. Vannavar Bush's Differential Analyzer in the year 1929 would sometimes take 2 months to calculate a differential equation. We consider that unacceptable.

An example of a terminal logical system would be AlphaGo, especially in its prototype stage. Go is a lot like a logical system and every once in a while AlphaGo would make a bad move but it would output an answer in a short amount of time. An
infallible system would be a handheld calculator with a maximum input of 8 digits. In very rare circumstances is it wise for a human to question its output and if a human should question it, the best way to do so would be to consult two other handheld calculators.

1.5. The Advantages of Natural Language Logic

1. Natural language logic is terminal in that for each set of symbols there is only one other set of symbols that it can be transformed into. Consequently, it is very easy to learn since it is quite easy for a human to test themselves as to whether or not they are transforming the symbols correctly. This is not to say that a reader cannot make changes to NLL if they disagree with it, rather if they do make changes then their system will also be terminal.

2. NLL is equipped to handle a wide variety of natural language constructions. Not all of these have been coded for yet but we still have made much progress in coding for relative clauses, adjectives, determiners, sentence properties and subordinate clauses.

3. NLL is written in natural language, consequently there is no confusion as to what a certain logical symbol refers to. Quite often when you ask a logician to pronounce their notation in natural language it is still not understandable. It is our hope that our symbols clearly correspond to natural language thoughts.

4. NLL currently calculates with roughly 50 definitions and uses roughly 200 distinct words but we have reasonable hypotheses for how roughly 500 other words are to be defined including all of the major metaphysical concepts such as truth, meaning, essence, probability, necessity, causation, reference, etc.

5. A logical system can only become persuasive if it can argue for a large number of intuitively true statements and NLL currently has argued for 180, though not all of them are obviously true. Each argument uses the same algorithm and consequently with the click of one button roughly 40,000 lines of argument are generated. The reader should not be turned off by the large number of lines, since many of those lines are irrelevant and others are just useful information so as to help the author understand what is going wrong should an argument fail.

6. NLL defines words in terms of a set of indefinable atoms. We do not merely define vague expressions with other vague expressions. All words are built up from a set of very easy to understand indefinables.
1.6. On Simplicity

Many philosophers hide their ignorance behind fancy language. Hegel, Heidegger, Derrida have been the most successful in introducing bizarre new jargon so as to pull the wool over the public eyes. Logicians are also quite guilty of this practice but it is much easier for them to get away with it because they are referring to abstract entities rather than real world phenomena. People are much more apt to accept the consistent, non-redundant existence of a new abstract entity then they are of a natural entity. We have absolutely no tolerance for philosophers who introduce new jargon for a word which already exists in the popular idiom. So 'the predicate \( b \) satisfies \( c \)' is just another way of saying \( 'c \text{ is } b' \). Or \( 'b \text{ is the value of variable } c' \) is just another way of saying \( 'c \text{ refers to } b' \). Or \( 'the variable b \text{ ranges over } c' \) also means nothing other than \( 'b \text{ refers to } c' \) or perhaps it means \( 'b \text{ refers to objects of kind } c' \). With our system we are actually going to take not multiplying entities beyond necessity with the utmost seriousness. The simpler our system is, the easier it will be for others to understand and the easier it will be to code for. For this reason, we dispense with all jargon which we believe is a mere synonym for a more popular word that the layman is already familiar with.

Further, I'm sure every scholar has gone through the experience where the author demands that the reader learn a huge list of rules. To make matters even worse, not only do they want you to learn new rules, they want you to figure out for yourself that they call things by different names or perpetuate the practice started by someone else. To my mind, this practice is unbelievably alienating to the reader.

We also have to remember that we logicians are in competition with the other sciences for brilliant minds. There are enormously intelligent teenagers out there and they want to devote their minds to the most challenging and satisfying problems. If we logicians just invent a bunch of new jargon for things which are already named, then they are going to be repulsed by our profession and will devote their immense talents to some other field. It is for this reason that we have very little use for numerous introductions of jargon.

1.7. Brief Remarks on Higher Order Logic

We reject the distinction between higher-order and first-order logic as uninteresting and a mere accident of history. It just so happens that for a very long time logicians had trouble formalizing a certain set of sentences but this does not mean that there is a very real meaningful difference between them. An analogy would be the difference between 890 cm and 835 cm. Bob Bemon smashed the then world record in the long jump by 55 cm and it took long jumpers another 22 years to break his record of 890 cm. The difference between 890 and 835 is a mere accidental of history, not some real psychological boundary that humans cannot cross.

That being said, our language is higher-order in that abbreviations can refer to anything. That is to say, they can refer to particulars, universals, relations, properties, groups, logical connectives, auxiliary words, quantifiers, even the negation sign. (An abbreviation is a constant or a variable). For example, the sentence: 'the quantifier "every" is an abstract object' can be formalized in this language. To my mind, higher-
order logic in the sense of Bueno "the main feature of higher-order languages is that they allow quantification over predicate and function variables" is too narrow. Suppose we develop a language which can predicate something about a predicate, we would then be stuck with an inability to predicate something of logical connectives or some other type of word, such as "the logical connective → is transitive". For this reason, higher order logic should be able to evaluate any sentence which first-order logic cannot. One can have a debate over whether or not some of our claims are higher-order. For instance, "it is⁰ contradictory that I ate an apple and apple is⁰ not an⁰ abstract⁰ term," does seem to be higher-order since we are predicating something of 'apple' which is not an individual but a universal and first-order logic can only make predications of individuals. In any case, we also falsify 'the⁰ property⁰ redness is⁰ red' which is certainly higher-order.

1.8. The Output/Decision Problem Distinction

I now want to preempt any confusion the reader might have as to what I am trying to accomplish. I am well aware that people are going to think I'm trying to accomplish something in metaphysics similar to what Hilbert was trying to accomplish in mathematics: a decision procedure which would determine the ABSOLUTELY TRUE solution to any mathematical theorem. All we're trying to do is come up with a decision procedure which will output a consistency value regardless of whether or not that consistency value is what we believe or not. We call this the output problem. A search for a decision procedure for an infallible system we are going to call the decision problem or Entscheidungsproblem. Before we can solve the decision problem we first need to be able to even produce an output and any output will do. We can then worry about refining our system such that we get the correct outputs later.

As I said earlier, we want a terminal system, not an infallible system. Without a decision procedure the logician wanders about blindfolded in the night-forest, rudderless in the wroth flash-ocean and is free to use any statement they like so as to show that a consequence follows from a premise. Or as Bradley put it: "metaphysics is the finding of bad reasons for what we readily believe upon instinct". Currently, with natural language there is no solution to the output problem which is a state of affairs that we will now seek to correct.

1.9. When a Philosopher Must Abide by the Experimental Data

Our goal in metaphysics is to provide definitions of all the major metaphysical words which make the least amount of changes to the accepted conventions of word usage. In almost all cases the conventional usage of the metaphysical concepts result in contradictions. For example, it is conventional and contradictory to say both 'Santa Claus does not exist' and 'Santa Claus exists as an idea'. When the layman is asked which conjunct to give up they usually give up the former. We already know that the conventional usage of the word 'exist' results in a contradiction, we do not need a survey
to tell us that. It is the job of the metaphysician then to come up with some rules such that usage of this word results in fewer contradictions. Some of the conventions therefore will have to be abandoned but the important point is that we want to abandon as few of these as possible since it will make things easier to understand.

However, there are times when we do not know what the convention is because, one, we encounter the usage so rarely, two, each possible usage is consistent and, three, we cannot come up with any reasons why one is more correct than the other. In this situation it is best that the metaphysician take a survey since the results can sometimes be radically different from what we expect. For example, we have to craft a definition of 'any'. So suppose someone enters a room and I say to them 'take any computer in this room' and they proceed to take two computers. Have they done something that I have not permitted? I think they have but Dayal thinks they have not. But notice that, one, if we accept my rule or Dayal's rule it will be quite easy to build a consistent system with either rule and, two, we really cannot come up with reasons why one rule is better than the other. Although of course I believe my rule is correct I am uncertain to the extent that I will not be surprised if the majority believes Dayal's rule.

1.10. The Rule of Deference to Popular Opinion

1. If two mutually exclusive usages of a word both result in a high number of contradictions then it is the job of the metaphysician to eliminate this contradiction.

2. If two mutually exclusive usages of a word do not result in a high number of contradictions and, one, it is not clear which usage the public prefers and, two, there are no reasons why one is more consistent than the other, then the metaphysician must take a survey and abide by the results. If the survey results are quite close to 50/50 then the metaphysician can simply put two entries into their dictionary which refer to each possible usage.

Let me give some more examples of 2:

3. Is the number two a non-professional?
4. If I say some men do not own rocks from Pluto do I assume that there is a man who does own a rock from Pluto?
5. If a sentence is negated and it has both the word 'all' and an adverb then is 'all' or the adverb negated? For example, in 'it is not the case that all of the Beatles spoke quickly to the green Martian' is 'all' or 'quickly' negated?
6. Can there be a group without members?
7. Can someone coexist with themselves?

Notice in these above five questions that no matter what we decide the effects of the decision are going to be remarkably slight and it will be quite easy to remove any other contradictions that our answers to these questions will cause. Also, note that we cannot really come up with reasons why the number is or is not a non-professional, for example. The answers depend on how you use the prefix 'non', and the words 'some', 'not', 'group',
and 'coexist'. These words are very foundational and their correct use is basically axiomatic.

8. Is a particle alive?
9. Can I observe a mind?
10. Can I sense an idea?
11. Can I experience a fact?
12. Can a fact exist in the imagination?

Here we actually can come up with some reasons for why they are true or false. Conducting a survey is not going to tell us why they are true which is what we are really interested in. For example, let's make a caveat that when we ask the above question we stipulate that there is at least one accepted literal usage of the words which makes the affirmative of the question consistent. If we allow for metaphorical usages then virtually every grammatical statement is consistent. So in 10 using the most common use of 'sense', it is false, but in a less common use it is true. After making that stipulation I'm going to guess that the percentages of those that answer 'yes' would be: 8 10%, 9 10%, 10 90%, 11 90%, 12 60%. It is preferable but not necessary that our system be consistent with these results, if the results were real. It could be that if we really think things through we will find out that our usage of the word 'alive' leads to all sorts of contradictions and the only way we can resolve them is to come up with a definition which answers 'yes' to 8.

1.11. How this Book is Organized

Our goal is to eventually formalize all the important areas of metaphysics. To do this what must be done is first make a superficial survey of these areas. You can't get bogged down in one little corner of metaphysics, otherwise the project runs the risk of being abandoned. So this current work spends 12,000 words just on the words 'a', 'no', 'every', 'the', and 'many' but whole books have been devoted just to the word 'a'. If we were to go into that much depth early on we would never get finished. Thus, after we make our first superficial survey, we will then make a more in depth survey. During the first superficial survey our goal is to cover a lot of ground quickly and consequently we do not have time to consider the ideas of other or tackle a lot of problem, that will be for the second survey.

1.12. Why there are no Arguments

All of the arguments are placed in an Excel file. Currently, the arguments take up 1200 pages but this because they are automated and I print out as much information as possible so that I can make sure every detail has been accounted for. I have not yet written up code which only prints out the essential information. After one begins to understand how the arguments work they are only going to pay attention to bits of and pieces of them and
are certainly not going to read all of the arguments but just check that they generate the right output then quickly scan over them for outrageous errors.

1.13. Relationships are Synonymous with Propositions

Before we get started there is some strange jargon that I have invented which is bound to confuse people.

Propositions are awkward terms. Even Alvin Goldman stated: "I regard propositions as a temporary theoretical posit from which we should ultimately ascend to a better theory." Stephen Kleene gives the standard understanding of propositions: "John loves Jane' and 'Jane is loved by John' express the same proposition." This does not tell us what propositions are just merely that they have a relationship to sentences. We are going to call propositions relationships since each proposition is composed of one relation and two relata. If only one relatum appears in the sentence then it is because the relation is an implicit relation which, when reduced, expresses the implied relatum. So 'this exists' reduces to 'this is extant', where 'is' is a relation. And 'something happened' reduces to 'this is extant at time 2 and it was not extant at time 1'.

1.14. Detached, Attached and Hypothetical Sentences

A detached sentence exists by itself on a line. So in the following set of premises:

1. p
2. p → q
3. q
4. s & t
5. ~(v & w)

p and q on lines 1 and 3 are detached. We do not have much use for attached sentences since these include sentences whose main connectives are &, hence 2, 4 and 5 are considered attached sentences. But 2 and 5 are what we call hypothetical sentences. That is a sentence whose main connective is either ≡ → ⊻ ∨ # or is the negation of a conjunction and we call these connective the hypothetical connectives. (The ⊻ and # sign will be explained in section 7.4. and 7.5. respectively). We use the word hypothetical because we are making a hypothesis that p and q have a certain relation to each other. We are not asserting that they have a certain truth-value.

Just as the sentences in 2 have been named antecedent and consequent we will also regularly refer to the sentences in p ≡ q. We call p the biconditional antecedent and q the biconditional consequent. Even though p ≡ q can be reversed without change of truth-value, we nevertheless use ≡ in definitions and always put the definiendum on the left and the definiens on the right. In the disjunction p ⊻ q we call p the disjunctive antecedent and q the disjunctive consequent.
2. The Rules of Natural Language Logic

2.1. Rules for a Grammatical Sentence

A sentence can be evaluated whether or not it's natural or artificial. If the sentence is natural then it must be grammatical but we do not yet have the rules for grammaticality spelled out. Moreover, when we eventually list the rules, they are not going to be the rules which prove whether any collection of English words is grammatical or not, rather only those that have syntax which we can evaluate. It is difficult to state precisely exactly what type of grammar we can process so I will just state some rough rules of thumb:

We can evaluate sentences where
13. one adjective modifies a noun.
14. one noun appears in apposition to another noun such as 'the philosopher Leibniz thought, but not 'Leibniz the philosopher thought'.
15. one relative pronoun such as 'which' or 'who' appears.
16. the coordinator 'and' appears as in 'Leibniz and Aristotle thought' but I only coded this for the subject position.
17. one prepositional relation appears such as 'in' in the sentence: 'Leibniz thought in Hanover'.

Some very common constructions which we cannot evaluate are sentences where
18. the word 'that' is followed by a phrase such as 'Leibniz thinks that this is good'.
19. the infinitival 'to' is used as in 'Marilyn wants to go home'.
20. there is a sentence property such as 'it is interesting that Kiera Knightley acted'.
21. there is a gerund such as "I saw Audrey Hepburn sitting".

2.2. Rules for a Well-Formed Formula

As the computer translates a natural sentence into an artificial one, the reader needs to know if a mistake has been made. The only sentence that need obey the grammatical rules of natural English are the initial premises. After that grammaticality is no longer required. Technically, there is a distinction between sentences and relationships but in this section when we say 'sentence' we really mean 'sentence' or 'relationship'.
22. If a sentence b is natural and grammatical and has redundant words and another sentence c has all of the words that b has in the same order except for the redundant words then c is well-formed.
23. A sentence is abbreviated in the following ways: all adverbs, adjectives, nouns are abbreviated with a single letter. So as to avoid confusion we say that relations are shortened and nouns, adjectives and adverbs are abbreviated. Relations are shortened as specified in our dictionary. If a certain word b has already been abbreviated in a different sentence then b must be abbreviated the same way each time.
24. If a sentence b is well-formed and has no abbreviations and sentence c is the same as b except that all of c's nouns, adjectives and adverbs are abbreviated and all of c's relations are shortened, then c is well-formed.

Definition: Standard Sentence

A sentence is standard iff it is composed of only relations, and abbreviations which refer to nouns. It may also have negation signs and abbreviations may refer to an adjective but only if the adjective is the object of the J relation. Abbreviations may also refer to adverbs but only if they are in the object position of the V relation. It must be stressed that the relation need not be atomic.

We will now state in what positions the non-literal symbols may appear in a standard sentence.

The Abbreviations

With the exception of 'a', 'i' and 'l', all roman letters are abbreviations. (the letter 'l' is not used because it looks too much like the number 1). Abbreviations are divided into three categories: constants (also called definite abbreviations), indefinite abbreviations and general abbreviations (also called instantiable abbreviations). These three types are not distinguished from each other by the use of prime signs, subscripts or Greek letters but through other means which we will explain shortly. Though we'll probably find exceptions in the future, abbreviations can quite literally refer to anything and I mean literally anything even the negation sign, relations, logical connectives, etc.

Lettered and Numerical Subscripts

We only use numerical subscripts when we need more than 23 abbreviations, so b is no different from b₁ and both can refer to anything. We use lettered subscripts as a temporary device until we have anaphora automatically coded for. So when two abbreviations are subscripted with the same letter then they must be instantiated or replaced with the same abbreviation.

Greek Letters
We only make a very small use of the Greek letters \( \alpha \gamma \) when we write up a list of all the properties that an object has. So when we want to write down on a list that \( b \) has the property 'sees \( c \)', this is written as \( (\alpha \text{SEE}c) \). The reason why we do this is because if we are trying to determine whether or not \( b \) instantiates \( d \), then \( d \) and \( b \) must share properties and it is easier if we change the abbreviation in a sentence in which \( b \) resides to the same letter which is \( \alpha \). The \( \gamma \) is used if another object in the same sentence is indefinite. But we will explain this in more detail later.

**The \( \sim \) (tilde or contradictory negation sign)**

A \( \sim \) may appear before a single relation in a sentence or before an open parenthesis. It may not appear before an abbreviation. If the \( \sim \) appears inside the sentence \( b \) then it may not appear before the open parenthesis that encloses \( b \) and if the \( \sim \) appears at the open parenthesis that encloses sentence \( b \) then it may not appear before any of the relations which compose sentence \( b \). For example,

25. \( (b \sim R c) \& (b \sim R c \sim T d) \& (b R c \sim T d) \& \sim(b R c) \)

26. \( (b \sim R c \sim T d) \& \sim(b \sim R c) \)

The conjuncts in 25 are well-formed whereas in 26 they are not well-formed. If a sentence \( b \) contains a \( \sim \) and another sentence \( c \) contains each word that \( b \) has in the same order but \( \sim \) is absent then \( b \) and \( c \) are both abbreviated with the same letter except that \( b \) has a \( \sim \) before it.

**The \( \neg \) (Consistent Negation Sign)**

The \( \neg \) may appear before any abbreviation or relation and it may appear more than once in a single sentence. The \( \neg \) may also appear in a sentence which has \( \sim \) in it. The \( \neg \) may not appear outside the sentence in front of an open parenthesis. If a sentence \( b \) has a \( \sim \) and \( b \) is a relata in a different sentence then \( b \) will have a \( \neg \) before it. For example, to formalize: 'I do not believe that it is not the case that there are dogs', would be eventually formalized as follows:

27. \( (i \sim B \neg b) \& (b \leftrightarrow \text{there are dogs}) \)

The sentence: "I want to go home more than I want to stay here" is equivalent to the much more stilted "I want that I go home more than I do not go home". The latter will reduce to the standard form as follows:

28. \( (i \text{ WNM } b \neg b) \& (b \leftrightarrow \text{i gohome}) \)

The highly unusual sentence: 'a non-man did not speak to a non-mammal' would be formalized as:

29. \( (a \neg b \sim SP a \neg c) \& (b=\text{man}) \& (c=\text{mammal}) \)
The $\leftrightarrow$ Sign (Sentential Identity)

The $\leftrightarrow$ sign may only appear immediately to the right of an abbreviation which refers to a sentence. On the left of the $\leftrightarrow$ sign must be the sentence which the abbreviation refers to, like so: $(b \leftrightarrow c \text{ R } d)$. Because $\leftrightarrow$ is always the main connective there is no need to enclose $c \text{ R } d$ in parentheses. The $\leftrightarrow$ may also identify an abbreviation with a set of sentences like so: $(b \leftrightarrow (c \text{ R } d) \& (e \text{ Q } f))$.

The = Sign (Non-Sentential Identity)

The = sign is a relation between non-sentences. If a $\sim$ appears before the = sign then that means that substitution cannot be performed. Right now, we also use the = sign to show how a word is to be abbreviated and we are quite certain that that will always be the case. We also use the = to mean identity between individuals such Mark Twain and Samuel Clemens but we will abandon this as soon as we have a better theory.

If a word or sentence is abbreviated then it cannot appear within a definition under the scope of $\equiv$. What I mean by under the scope of $\equiv$ is that in the sentence $(p \& q \& ((r \& s) \equiv (t \& u)))$, $\equiv$ has scope over r, s, t and u but not p and q.

The Instantiation Sign $\Rightarrow$

Anywhere the abbreviation to the left of the $\Rightarrow$ appears it may be substituted for the abbreviation to the right of $\Rightarrow$ but not vice-versa. The $\Rightarrow$ sign may also have a variable R or Q relation on its left and another relation on its right. Although the following rule can be derived: a constant may not appear on the left of the $\Rightarrow$ sign, nor is the $\Rightarrow$ sign a transitive relation. No other symbols and no more than two abbreviations or two relations may appear within parentheses which contain $\Rightarrow$. The sentence which has $\Rightarrow$ is not used in the propositional calculus and it is called an instantiation sentence. If an instantiation sentence appears on a line then only other instantiation sentences or translation sentences may appear on that line. Consequently, none of the logical connectives $\& \rightarrow \equiv \text{ R } \text{ V } \#$ may appear on the same line as an instantiation or translation sentence.

The Translation Sign $\equiv$

All the rules which apply to the instantiation sign also apply to the translation sign except that a constant may appear to its left.

The Logical Connectives

The logical connectives $\rightarrow \equiv \text{ V } \text{ V } \#$ must be flanked by sentences. A second logical connective cannot come between one logical connective and a sentence. The connectives $\& \text{ V } \text{ V }$ may appear in sets greater than 2 like so: $(b \& c \& d)$ or $(b \text{ V } c \text{ V } d)$ or $(b \text{ V } c \text{ V } d)$ but this permission does not apply to $\rightarrow \equiv$ or $\#$. If a set of 3 or more sentences are connected without any parentheses between them then they must all be connected with
the same connective. Hence, \( (b \lor c \land d) \) is not well-formed. We do not use rules of precedence.

**The [period] Sign**

A period may come between two or more abbreviations, like so \((b.c \ R \ d)\) or \((b.c.d.e.f \ R \ g)\). It may also appear in the object position, like so \((b \ R \ d.e.f)\). A period may not appear in both the subject and object position like so \((b.c \ R \ d.e)\). And if a period appears in either the subject or object position then each abbreviation in the position must be separated by a period. So \((b.c \ d \ R \ e)\) would not be well-formed. It also does not matter whether or not these abbreviations are negated so \((b.\neg c.d \ R \ e)\) is well-formed.

**The Inference Sign** ⇐

The ⇐ sign is only used in two sentences and can be used nowhere else. It is the main connective.

\[
((p \land \neg q \land J c) \land (p \lor d) \Rightarrow (q \lor d)) \land (b=\text{consistent})
((p \land \neg q \land J c) \land (p \lor d) \Rightarrow (q \lor d)) \land (b=\text{improbable})
\]

**The Connective Reduction Sign** ⇔

The ⇔ is only used in the definitions of the logical connectives. It is the main connective.

**The Justified Conditional Sign** ⊢

The ⊢ sign acts just like the → sign except that it can be argued that the consequent follows from the antecedent. The ⊢ may not appear in a definition, a partial definition or an axiom.

**The Justified Biconditional Sign** ⊣⊢

The ⊣⊢ sign acts just like the ≡ sign except that it can be argued that the biconditional consequent follows from the biconditional antecedent and vice-versa. The ⊣⊢ may not appear in a definition, a partial definition or an axiom.

**The Consistency Sign** ⊗

The ⊗ sign can only appear as the last line of an argument. Further, the penultimate line must be a conjunction where no sentence is both has and does not have the \(\sim\) in front of it.

**The Contradictory Sign** ⊥
The \( \bot \) sign can only appear as the last line of an argument. Further, the penultimate line must be a conjunction where the same sentence has and does not have the \( \sim \) in front of it.

**The Contingent Identity Sign \( \backslash \)**

In a definition or axiom all the abbreviations must be different which means they cannot be on both sides of the \( = \) or \( \Rightarrow \) sign. If it is contingent but not necessary that two abbreviations are identical or one is instantiable by the other within the same definition then they are separated with the \( \backslash \) sign, like so \( (b \backslash c) \). Only two abbreviated relata may appear in a sentence with the \( \backslash \) sign.

**Mathematical Signs**

We use \( / \) for division, \( * \) for multiplication, and \( + - = \) as they are normally used in mathematics. The order of precedence is \( = / * - + \). We do not place these signs between actual numbers but only their abbreviations. Hence, \( 2 + 2 = 4 \) would be \( (b + b = c) \& (b=2) \& (c=4) \).

**Relations**

A sentence may contain a high number of relations. The relations R and Q are instantiable. For example, a sentence with the CA relation, 'cause', contains 9 relations: \( (b \text{ INM} c \text{ E} d \text{ U e T f CA g INM c E h U e T j}) \) and is pronounced as: 'relationship b in\(^m\) [region] c in\(^r\) [the actual world] d in\(^u\) [universe] e at\(^t\) [moment] f causes [relationship] g in\(^m\) [region] c in\(^r\) [the actual world] h in\(^u\) [universe] e at\(^t\) [moment] j'. The first relation in a standard sentence must appear after a relatum, so \( (R c) \) is not well-formed. But in a natural language sentence the first word can be a relation such as 'in a town where I was born there lived a man who sailed the sea.' No more than two relations may appear side by side. So \( (b \text{ OC T d}) \) is well-formed but not \( (b \text{ OC S T d}) \). Only the first relation may exist without an object. So \( (b \text{ OC T d}) \) is well-formed but not \( (b \text{ OC c T S d}) \).

**The Order of Prepositional Relations**

The prepositional relations state where a relationship is true or consistent and by where I mean space, time, possible world, possible universe and language. To explain what I mean by 'language': the relationship "Russell is dressed smartly" is consistent in:\(^1\) the language British English but is contradictory in:\(^1\) the language American English. The one exception is 'of' which does not state where a relationship is true or consistent and appears as a prepositional relation. Below is a list of the most commonly used prepositional relations and the order in which they appear in a relationship:

**Spatial**

INM - a group of particles is in\(^m\) INM a group of points.
IN - a group of particles is in IN another group of particles.
S - a single particle is at S a point.
ON - a group of particles is above ON another group of particles.

Possible World:

P - a thing is at P a possible world.
E - a thing is in E the possible world which is actual (E was chosen arbitrarily).

Possible Universe:

U - a thing is in U a possible universe.

Temporal:

T - a thing is at T at a moment in time.
DUR - a thing exists during a group of moments.

Language:

IL - a relationship exists truthfully in L a language.

It is true that I exist in m San Diego at P possible world c in U possible universe d during 2017 in l English.
(b=san diego) & (c=2017) & (f=English)
(i EX INM b P c U d DUR e IL f)

'O' spelled OF can exist in between any of those prepositional relations:

I exist in m a part of San Diego.
(b=part) & (c=san diego)
(i EX INM a b OF c)

Multiple Place Relations

If a relation relates more than two relata then these must be separated by a space. So (b R c d) is well-formed but not (b R cd). Two relata may also appear in the subject position without there being an object as in the sentence: 'Harry and Sally met' which is formalized as (h s MT). Some relations have a high number of relata to their right. The move relation has four: (b MV c d e f) and is pronounced as "b moves from c at d to e at f".

The Division of Argument

Our arguments are divided into sections.
Section One: composed of premises and these premises may be in natural language or artificial language.

Section Two: composed of the identity statement, untranslated premises or lemmas, and instantiation or translation sentences.

Section Three: composed of translated premises or lemmas.

Section Four: composed of inferences made from section three.

Section Five: composed of sentential abbreviations.

Section Six: composed of the premises and inferences from sections one, three and four, but this time each sentence is abbreviated with a single letter.

One rule that we should point out here is that single abbreviations of sentences may not appear in section three or four between logical connectives. We call sections one through four, relational logic and section six is propositional logic. There can be no mixture of the two. So in propositional logic logical connectives come between single letter abbreviations of sentences whereas in relational logic connectives come between sentences which contain at least one relation and one relatum.

30. \((b \text{ R } c) \& d \& (e \text{ Q } f)\)

30 would not be well-formed because d stands for a sentence and hence is propositional logic whereas \((b \text{ R } c)\) contains a relation and at least one relatum and hence is relational logic.

**Conventions**

If we force the logician to obey numerous rules regarding well-formed formulas then this will slow them down. On the other hand, if our rules are too lax then this will make things harder to read. Hence, we reach the following compromise: in addition to unbreakable rules, we also have conventions which logicians are expected to follow but do not have to for the purposes of making arguments cleaner and easier to read. In truth, however, the following conventions are very easy to program for so the logician will not even have to voluntarily follow them.

31. The relata, relations, the signs ~, ⇔, logical connectives and mathematical signs in a definition, partial definition or axiom should be separated by a space.
32. A space should not come between a period and the relata, nor the \(\setminus\), ⇒, ≍, = signs and their relata. A space should not come between the ~ and the relata or relation that it negates.
33. Parentheses should not enclose a sentence if it is abbreviated with a ⇔.
34. The abbreviations in our definitions should begin with the letter b and progress orderly through the alphabet. This is very hard to do when writing a definition but we have software which fixes this problem.

35. When we define a noun or adjective, the first sentence in our definition is of the form (b=word), where 'word' is whatever word is to be defined currently. We then write out our definition. In the penultimate position we place relationship abbreviations and in the ultimate position we place the abbreviations of words.

36. When an adverb is defined it is done as a property of a sentence. For this reason, the adverb to be define is placed first just like in a noun or adjective, then the sentence that it is a property of, then the definition proper.

37. If a definition is so large that it becomes too difficult to read, the definition should be split up over several lines with obvious breaks.
3. The Metaphysical Atoms and Molecules

3.1. The Common Metaphysical Atoms

We define words with other words. If we keep defining words with words and if our vocabulary is finite then we are eventually going to encounter the indefinable words. We lump these indefinable words into two categories: relations and non-relations and the non-relations are mostly properties. The indefinable relations are:

After, at, desire, has, in front of, is, left of, believe, above

All of these words have various meanings, so let's now disambiguate them. The most important thing to understand is how the atoms are pronounced since a large chunk of every argument is composed of the atoms. Also, henceforth we enclose natural language words in [ ] to mean that nothing in the artificial notation corresponds to these words but they are nevertheless pronounced when reading the notation out loud since they can be inferred.

(b A c) - b [is] after c
(b D c) - b desires c
(b H c) - b has c
(b I c) - b is$^g$ c
(b B c) - b believes c
(b L c) - b [is] left [of] c
(b F c) - b [is in] front [of] c
(b AB c) - b [is] above c

Because A is taken, we use the S relation to express 'at'. (The S was chosen because 'space' begins with S):

(b S c) - b [is] at c

Because A is taken, we use the T relation to express 'at$t$' in the sense of 'at a moment in time':

(b T c) - b [is] at$t$ c
3.2. The Metaphysical Molecules

We are not yet done explaining all of the metaphysical atoms but before we can explain any of them further it will help if we start talking about the molecules. In this system, at all times we have to have a definite belief about what category our abbreviations belong to. For this reason, only objects of a certain category may stand in certain positions of the indefinable relations. The metaphysical molecules are defined solely in terms of where they stand in relation to an atomic relation. For example, 'moment' is always the object of the T relation, 'point' is always the object of the S relation, and 'universal' is always the object of the I relation. We can now explain the atomic relations in greater depth:

(b A c) - [moment] b [is] after [moment] c

When writing down the sentence (b A c) we need not state that b and c are moments because that information can be inferred.

(b H c) - [thing] b has c
(b I c) - [instance] b is® [universal] c
(b L c) - [point] b [is] left [of point] c
(b F c) - [point] b [is in] front [of point] c
(b AB c) - [point] b [is] above [point] c

Because 'instance b is® universal c' is slightly awkward we also sometimes say 'b is an instance of c' which translates to (b I c).

3.3. Negative Categories

In some cases, the category to which an object belongs is negative, meaning that the necessary and sufficient condition for belonging to that class is that it not belong to a certain class. So a 'non-possible world' is any object which is not a possible world, a 'non-moment' is any object which is not a moment. In the popular speech, many words which have the prefix 'non' do not have this property. So a 'non-professional' is also a human and hence it is a contradiction that the number 2 is a non-professional. When we define such words as 'non-professional' we would have to put in its definition that it is at least a human.

3.4. More explanation of the Molecules and Atoms

(b D c) - [mind] b desires [relationship] c
(b B c) - [mind] b believes [relationship] c [is true]
Because the A relation is taken we use G to mean 'after' in the sense of number:

\[(b \ G \ c) - \text{[number]} \ b \ [is] \ \text{after}^n \ [number] \ c \] (here, 'after' is synonymous with 'greater than' which is why G was chosen)

Because the T relation is taken we use the C relation which relates a mind to a thing and it is pronounced as 'think about'.

\[(b \ C \ c) - \text{[mind]} \ b \ \text{thinks about} \ [thing] \ c \]

C was chosen because 'think about' is synonymous with 'contemplate', though 'contemplate' sometimes implies that one is doing the contemplating peacefully. The difference between the B and the C relation is that if I think about b then it does not follow that I believe b is true. Further, I can think about a rock whereas I cannot believe a rock, that is to say, anything can be the object of the C relation.

Because the I and D relation are taken we use the J relation to relate a thing to a property in adjective form:

\[(b \ J \ c) - \text{[thing]} \ b \ \text{is}^\alpha \ [property] \ c \]

Take the sentence: 'Leibniz is courageous iff Leibniz has courage'. Because we believe the words 'is' and 'has' are different and because 'courageous' and 'courage' are considered different words, we thus use the H relation to relate a thing to a property in noun form:

\[(b \ H \ c) - \text{[thing]} \ b \ \text{has} \ [property] \ c \]

Because A has already been taken we use the P relation to relate a thing to a possible world:

\[(b \ P \ c) - \text{[non-possible world]} \ b \ [is]^{\theta} \ [possible world] \ c \]

The object of the T relation is always a moment:

\[(b \ T \ c) - \text{[non-moment]} \ b \ [is]^{\iota} \ [moment] \ c \]

The object of the S relation is always a point and the subject can either be an indivisible unit of space or a particle:

\[(b \ S \ c) - \text{[particle]} \ b \ [is] \ [point] \ c \text{ or } \text{[void]} \ b \ [is] \ [point] \ c \]

Now, suppose a critic were to ask "why can't a possible world be the subject of the P relation. Can't a possible world exist in itself?" Recall that we are only here to explain how the notation works. We will provide arguments for the irreflexivity of the P and T relation at a later point.
3.5. The Distinction Between the W and I Relations

Because the H, G and S relations have already been taken, we use the W relation to relate a group to a member since it is the first letter in the word 'whole'. Further, we make no distinctions between 'wholes', 'groups' and 'sets', nor do we distinguish between 'members' and 'parts'. We do however make a distinction between 'groups' and 'universals'. First, we are not willing to deny that there is a part which is not part of a whole. Second, after an extensive analysis of several uses of the word 'part' and 'member' we found no occasion where it would be logically impossible to replace one for the other. Certainly, there are cases where it is awkward, for example, 'the steering wheel is a member of the car' is certainly strange but I do not believe it to be logically impossible. For this reason, we make no distinction between 'parts' and 'members' and since we do not distinguish between these two, it would be contradictory to distinguish between wholes, groups and sets.

There is a distinction, however, between 'this is a tiger' and 'this heart is part of this tiger'. Suppose the tiger is named Felix. We do not point at Felix's heart and say 'this is a Felix'. In other words, there is a distinction between named groups and unnamed groups. For the members of named group b it is consistent to say 'this is a b', but for the members of unnamed group c it is not consistent to say 'this is a c'. Hence, the subjects of the I relation are named whereas the objects of the W relation are not named. What this means is that (b I c) is entailed by 'b is a c', but (d W e) does not entail 'e is a d'. (With the I relation the universal is the object, and in the W relation the group is the subject. This is because W is pronounced as 'has' and I is pronounced as 'is').

3.6. The Difference Between Concepts, Universals and Arbitrary Groups

We are going to make an exception to our rule of not going too deep into the details which do not specifically pertain to the rules of our notation. For a very long time we were unable to make any distinction between concepts and universals. In reviewing a list of the 300 most common nouns which are not proper names, each one is both a universal and a concept. So probably more than 99% of all universals are concepts, even though a finite list of all universals is very hard to produce.

We divide the referents of all words into two categories: universals and particulars. We humans simply cannot give every distinct object a different name, so we put many objects into the same category b which allows us to then utter 'this is a b'. Now the distinction between concepts and universals is as follows: just because a small group of people decide to name their group is does not follow that they have invented or discovered a concept. For example, it is not conventional English to say that the names of music groups or the names of sports teams among others are concepts. For example, it is not conventional English to say that the names of music groups or the names of sports teams among others are concepts. So we simply do not say that 'Beatle' or the baseball team 'Yankees' are concepts. However, all those groups which are named do belong to a category distinct from unnamed groups and we have decided to call named groups 'universals' even though this is not how many philosophers would use the term.
The way we draw the distinction between universals and concepts in noun form therefore is that those named groups which exist in reality solely because the members of the group itself or someone else decide to belong to the group are not concepts. So the only reason why the Beatles once existed in reality is because John, Paul, George and Ringo decided to be a group. The only reason why a certain group of prisoners, for example, belong to a group is because the prison guards decided to put them in a group and name them so as to make communication between the guards easier, but it does not follow that this group is a concept. The only reason why a set of tables at a restaurant are named is so that waiters can easily communicate amongst each others who serves whom, but it does not follow that the waiters by using this name have invented or discovered a concept.

3.7. The N Relation

We use the N relation to count the members of a group. Right now, the subject of the N relation is a group and the object is a number but we might reverse this in the future. The N relation is pronounced as 'instantiate' and spelled with an n superscript. This is because numbers are properties just like any other. Further, there is a paradox which goes back to Quine [quote] and has now been worked on by roughly 10 philosophers all with different solutions:

38. The number of planets is 8.
39. It is necessary that 8 is greater than 6.
40. Therefore, it is necessary that the number of planets is greater than 6.

This paradox is solved because 38 is pragmatic for 'the set of planets instantiates\(^n\) the number 8' or 'the planets instantiate the number 8. Sets instantiate numbers just like non-sets instantiate properties.

3.8. The Obscure Metaphysical Atoms

The aforementioned relations make up more than 95% of all the atomic relations which appear in a premise. We are now going to talk about the more obscure atomic relations.

(b AL c) - [letter] b [is] after\(^l\) [letter] c

When we get around to doing more work on the mind and the distinction between 'real' and 'imaginary' we will use the following relation:

(b M c) - [thing] b [exists] in\(^l\) [imagination] c
When we ask where is a certain pain, it does not make sense to point to a location in space, rather it exists in some private space. We use the O relation to refer to this notion:

(b O c) - [sensation] b [is] ato [point] c.

The O was chosen purely at random. The Z relation is only used for geometry and its object is a point in two-dimensional space. Z was chosen at random.

The V relation relates a thing to a property in adverbial form. The motivation behind the V relation is as follows: we need to have a belief as to what category each object belongs to. So when we ask what category do the words 'always', 'now', 'necessarily' belong to, I think the best answer is that they are properties but properties in adverb form which is to say they are different from properties in adjective form such as 'red', 'good' and 'big'. So when we relate the relationship p to the adverb 'always' we do so using the relation V. V was chosen because it is the third letter in 'adverb'.

We also have to have a belief about what category the words 'the', 'a', 'many', 'few' belong to. Linguists will say that they are determiners but I think they are actually properties. So 'the man b walked down the street' means that 'b walked down the street and b is an instance of man and b has the property "being definite"'. We use the Y relation to relate things to properties in determiner form. (Y was chosen arbitrarily)

We also have the extremely rare U relation. We make a distinction between possible worlds and possible universes. For example, how would you express that it is possible that possible world b is actual at time 1 and possible world c is actual at time 2? To resolve this we assert that both b and c exist within something larger which we call a possible universe. Further, how would you express the fact that in our universe the laws of nature do not change? For instance, light travels at 299,792,458 m/s. How would you express the fact that light cannot travel 400,000,000 m/s in our universe but perhaps there is another universe in which light can travel at 400,000,000 m/s. Again, to solve this problem we use the U relation which relates a set of possible worlds to a possible universe.

Next, there is the IL relation. This relation is only used to handle very obscure paradoxes. So how do you express the fact that in 1971 in British English a billion meant 'had 12 zeroes' but in 1972 in British English a billion meant 'had 9 zeroes'. You cannot say that 'in 1971 it was true in all possible worlds that a billion had 12 zeroes' and 'in 1972 it was true in all possible world that a billion had 9 zeroes' because what exists in all possible worlds does not change. For this reason the IL relation relates a relationship to a language. (IL stands for 'in language').

Finally, the AR relation is only used for defining the molecules and a few other words. When we look at the sentence (b A c), the relatum b, the relation A, the relatum c are not letters but non-letters. For this reason, when we say that relation A is after relatum b we cannot use the AL relation but must use the AR relation. (the A stands for 'after' and R stands for 'relatum' or 'relation').

### 3.9. Atomic Non-Relations

We now need to discuss 8 atomic non-relations which are mostly properties:
real, extant, consistent, grammatical, energy, here, now and many

'Real' is a property of a possible world. If we want to use 'real' as a property of a non-
possible world we have to use a different meaning of the word which has 'real' in its
definition. The only thing that I want to stress about 'real' at this point is that it has a
relationship to 'desire' which 'imaginary' does not have. So if I desire at time 1 that
something be real at time 2, it does not follow that it will be real at time 2. Since 'desire'
is an atomic relation, this relationship must be taken as axiomatic. If I desire at time 1
that something exist in my imagination at time 2, then it follows that it will exist in my
imagination at time 2. Now, suppose a critic were to say: 'what about tinnitus? It exists
in my imagination and I cannot desire it to go away.' In this system, tinnitus would be a
type of sensation. Drawing the distinction between imaginary objects and sensations will
be worked out later.

'Extant' was chosen rather than the more common 'exists' or 'existence' because we
chose arbitrarily that properties in noun form are less basic than properties in adjective
form. By basic I mean that in our decision procedure all non-basic symbols are converted
into basic symbols. So 'white' is more basic than 'whiteness' and we try to take the more
common word as more basic. Usually, properties in adjective form are more common but
with 'existence' this is reversed.

It is very important to point out that in this system, if I think about something then at
the very least it exists in the imagination but of course it does not follow that it exists in
reality. So everything that we talk about exists in some way. Further, when the layman
says 'ghosts do not exist', in this system, that is a pragmatic sentence which is a
paraphrase of 'ghosts do not exist in reality'. The most important axiom regarding
existence that we want to stress at the moment is the following:

41. If b does not exist in c then b does not have any properties in c.

So whenever we talk about 'nonexistence' it is only nonexistence in a certain space. As
far as the atom 'energy', we must point out that we're not here to do particle physics.
Particle physics has a whole set of words which are not defined in terms of other words
such as 'spin', 'mass', and all the fundamental particles which as of 2017 is counted at 17.
We only use one of these atomic expressions 'energy' so as to distinguish particles from
voids in space. While it is true that some physicists believe that space has energy we do
not believe this. This is not something that is measurable or observable but it is an
inference made with logic and when the physicists use logic on objects which cannot be
measured with instruments they are no more expert than we are.

'Grammatical' I think is for the most part understandable. Our system at present has
no grammar whatsoever, but we would eventually like to be able to state which sentences
are grammatical and which are not and when we do, we need to use the word
'grammatical' to express our rules for what counts as a grammatical sentence.

There are times when humans cannot count the number of objects but we can at least
look at two sets and say that this set has more members than this other set. Similarly, we
might look at 100 sets, say, piles of rocks, and although we cannot count the rocks in the
sets we can at least put them into five categories and say that members of the fifth
category constitute 'very many', members of the fourth constitute 'many', members of the third constitute a medium size, members of the second constitute 'few' and the final set constitute 'very few'. We take the word 'many' as basic and define the other words in terms of it.

'Here' and 'now' I think need no explanation but consistency I think requires extensive explanation.

### 3.10. On Consistency

Although it is highly controversial, for argument's sake, let's assume that language's chief purpose is to rule out possibilities. So when I utter to b 'p is true', although b has not ruled out 'p is false', if b believes I'm not lying then b has ruled out 'b does not believe that p is false'. Suppose further that you're dissatisfied with English's vagueness and you want to build an exact language utterly from scratch so that your interlocutors can have no doubt as to what other beliefs you might have if you utter one sentence. There are three words you need first to explain all the other words, these are 'grammatical', 'consistent' and 'true'. (Right now we are conducting a psychology experiment where we explain a foreign language to someone using only the words 'grammatical', 'consistent' and 'true' and ostension but the results are not in).

The first thing I have to do is I have to explain to my interlocutor that I'm going to use certain symbols and certain noises and they are only allowed to appear in a certain order. To explain this I'm going to use the word 'grammatical'. I'm going to say that certain word orders are grammatical and other word orders are not grammatical which is to say, those word orders are never going to appear in my language. This has not allowed me to rule out any possibilities yet. So one now knows that 'this is red all over' is grammatical but if they do not know the meaning of 'green' then they cannot rule out the possibility that 'this is green all over'. To rule out possibilities through the meaning of words alone we need to resort to consistency.

When I build my language I'm going to go through in a systematic way which two sentences cannot be believed at the same time, even though both sentences are grammatical. This is where I'm going to use the word 'consistent'. Definitions really are sets of four conjunctions where we specify what can and what cannot be believed at the same time. Using a toy definition of triangle, when I build my language and come to the word 'triangle' I will state the following two conjunctions cannot be believed at the same time, using the word 'consistent':

1. It is not consistent that b is a triangle and b does not have three sides.
2. It is not consistent that b is not a triangle and b does have three sides.

And that the following conjunctions can be believed at the same time:

3. It is consistent that b is a triangle and b has three sides.
4. It is consistent that b is not a triangle and b does not have three sides.
So what consistency does is it enables the listener to rule out certain possibilities. When I utter that "this is a triangle", my interlocutor can immediately rule out the possibility that "this has four sides".

It is very important to point out that our logic calculates consistency and contradiction. Hence the following conjunction: b & ~c & d & ~e & f & g is considered to be consistent since the same relationship is not affirmed and denied. We do not calculate contingent truth and falsehood. But because we can calculate consistency and contradiction we can also calculate necessary truth since b is necessarily true iff b is consistent and not b is contradictory.

3.11. Atomic Non-Literals

Our system has 9 atomic non-literal symbols:

\&, ⇔, ⇒, =, ¬, ≍, ⇢, ⇔

It is very important to point out that & in our system does not mean the standard truth-table that we are all familiar with:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>&amp;</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

We are not here to calculate contingent truth and falsehood but consistency and contradiction, or we are determining whether or not a set of sentences can be believed at the same time. As De Morgan pointed out and then proceeded to ignore his own advice: '[Logic] simply takes care that the inference shall certainly be true, if the premises are true.' We take that to mean that every premise in our logic is true even if the word 'not' or 'false' appears in the premise. So if our premise is 'it is false that there are dogs' what that really means, 'it is true that it is false that there are dogs'. If our premise is 'there is not a dog' that really means that 'it is true that there is not a dog'. Hence, p & ~q in our system is a true statement. This point is of the utmost importance because as logicians it is not our job to walk about in nature and determine what is contingently true and false, rather whether or not a set of putatively true sentences can be believed at the same time.

So if & does not mean the truth-table referred to above, then what does it mean? In this system we do not give reductive explanations of what atoms are. The slogan is: do not ask what an atom is but how it is related to the other atoms. The & sign has a relation to 'consistency', the ~ sign and the other molecular symbols →, =, ∨, ∨ and # are defined in terms of it. When we explain the symbols → and =, etc, we will make another attempt to explain it.

The = symbol in (b=word) really means 'b is an abbreviation of 'word'. We have ambitions to not use = to stand for 'is' in the sentence:

42. Mark Twain is Samuel Clemens.
We believe that 42 can be analyzed as:

43. \( b \) is an intrinsic property of Mark Twain iff \( b \) is an intrinsic property of Samuel Clemens.

And then we would use instantiation or the \( \Rightarrow \) symbol to argue that Mark Twain wrote 'Tom Sawyer' iff Samuel Clemens wrote 'Tom Sawyer'. But we cannot yet draw the distinction between intrinsic and extrinsic properties so we continue to use the \( = \) symbol.

The \( \Leftrightarrow \) symbol has the same meaning as \( = \) it's just that its relata are sentences not non-sentences. This is because \( (p = (b = c)) \) is confusing whereas \( (p \Leftrightarrow (b = c)) \) is not confusing. Also, \( \Leftrightarrow \) is always the main connective so \( (p \Leftrightarrow (b = c)) \) can be written as \( (p \Leftrightarrow b = c) \)

The \( \approx \) symbol in \( (b \approx c) \) means '\( b \) is translated into \( c \)'. To understand how this works, suppose we were to use the \( = \) symbol for the translation of German into English. We would then get:

\[
\begin{align*}
\text{kahl} &= \text{stark} \\
\text{stark} &= \text{strong}
\end{align*}
\]

So 'kahl' in German means 'stark' in English and 'stark' in German means 'strong' in English. Using the transitivity property of the \( = \) symbol we would get the false result that 'kahl' in German means 'strong' in English. In our system, when we define two words we have to put them in a static list. This means that we will use the same abbreviation for two different things, for instance, here is the lemma of moment (it's not a definition because the following can be proven from the definition of moment which is much more cumbersome):

44. \((bIc) \not\vdash (dAb)) \& ((bIc) \vdash (bAe)) \& (c=moment)

\( b \) is\( ^e \) \( [a] \) moment iff \( d \) \( [is] \) after \( b \) and \( b \) is\( ^e \) \( [a] \) moment iff \( b \) \( [is] \) after \( e \)

On the other hand, here is the lemma for 'in' in the sense of a set of particles is within a set of points:

45. \((bINMc) \not\vdash ((((bWd) \equiv ((dSe) \& (cWe))) \& (bWf) \& (fSg) \& (hWg)))

\[ \text{[set of particles] } b \text{ [is] in } ^m \text{ c is equivalent to [set] } b \text{ has } ^w \text{ d iff d [is] at [point] e and [set] c has } ^w \text{ e, further, [set] } b \text{ has } ^w f \text{ and f [is] at [point] g and [set] h has } ^w g \]

In 44 abbreviation \( b \) refers to a moment and in 45 abbreviation \( b \) refers to a set. This would result in a contradiction since sets cannot be moments. For this reason whenever we use the premises contained in our static list we need to translate the abbreviations into new abbreviations such that the same abbreviations do not refer to different objects. To do this we use the \( \approx \) sign.

Many logicians write their rules in the following form:

\( p \rightarrow q \)
The line that divides p and q is actually a symbol and it needs to be explained. First off we need to get clear about the fact that when we write relationships on a line, it follows that we either believe them or we are suspending our belief and simply determining what we would believe if we did believe them. So what the above modus ponens argument means is that if we believe \( p \rightarrow q \) and \( p \) at time 1 but have not asked ourselves if we believed \( q \), then if we do ask this question at time 1, then we must at time 2 believe \( q \). With modus ponens argument consisting of 3 relationships this would never happen, but when 20 or 30 such relationships are connected, it is not always clear what follows from them. This notion of deciding that we must believe something where formally we were not sure is what the line is meant to symbolize. Since our dictionary is written in a table format we cannot separate inferences by lines but instead use the \( \Rightarrow \) symbol. So \( (p \land (p \rightarrow q)) \Rightarrow q \) has the same meaning as the modus ponens argument referred to above. It should also be stated that if \( q \) existed by itself on a line, we could not write what is on the left on different lines.

Our system has one inference rule for necessary inferences and one inference rule for contingent inferences. Consequently, the \( \Rightarrow \) symbol is only used on two occasions.

46. \(((b \land c \land d) \land (b \neg c \land ~ J d) \land (b \land J e)) \Rightarrow (c \land J e) \land (d=\text{consistent}) \land (e=\text{true})
If b [and] c are consistent and b [and] not c are not consistent and b is true then c is true.

46 in simpler terms can be expressed as "if b and not c would result in a contradiction and b and c are consistent and b is true, then c is also be true." And the inference rule for contingent statements is:

47. \(((b \land c \land d) \land (b \neg c \land ~ J d) \land (b \land J e)) \Rightarrow (c \land J e) \land (d=\text{probable}) \land (e=\text{true})
If b [and] c are probable and b [and] not c are not probable and b is true then c is true.

Probability, however, is enormously complicated and we have not yet coded for it. We understand that this assertion that there are only two inference rules is highly controversial. Since we are only here to get the reader to understand our notation as fast as possible we will make arguments for these beliefs later.

It should also be stated that we do not consider conjunction elimination and introduction to be inference rules but mere organizational devices. So if we wanted to be very annoying we could have a logical system without it, it would just be very hard to read. As for whether or not instantiation is an inference rule, that requires an argument which will be made later.

The \( \Leftrightarrow \) is only used for defining the symbols \( \equiv \rightarrow \forall \lor \# \). We do not take \( \equiv \rightarrow \forall \lor \# \) as basic but can be defined in terms of \&, \\sim, \neg, J and 'consistent'. However, we cannot define \( \equiv \) with \( \equiv \), hence we use \( \Leftrightarrow \) to mean if what is on the left is written on a line then we may write what is on the right on a new line and vice-versa.
The reader may have noticed that we have used two different negation signs. The difference between these is very easy to state:

p & ¬p is not a contradiction.
p & ~p is a contradiction

To understand why, notice that not every set of sentences which share all properties except for 'not' are not contradictory:

48. Some men go to the beach and some men do not go to the beach.
49. Sometimes in March I drank coke and sometimes in March I did not drink coke.
50. I have a kidney and I have a non-kidney.
51. Mammals live on Earth and non-mammals live on Earth.
52. There is a woman who drinks and there is a woman who does not drink.
53. It is possible that Bell invented the telephone and it is possible that Bell did not invent the telephone.

In 48-53 each pair has all words in common except for 'not' and 'non', some auxiliary words and different verb declensions and they are not contradictions. To express this meaning of 'not' we use the symbol.

### 3.12. Molecular Non-Literal Symbols

We also use 11 molecular non-literal symbols which can be defined in terms of the basic symbols. These are:

⊥, ⊆, [period], ⊢, ⊣, ⊤, ⊥, ⊨, #

⊥ simply means not consistent and ⊥ means consistent. We used to use ⊤ to express consistency but ultimately we decided that this is wrong because ⊤ is most often used to mean tautology and in this system tautology means logically necessary which is different from consistency. The [period] is simply an abbreviation device used to make our definitions easier to read. It's meaning is as follows:

\[(b \text{ ATE } c) ≡ ((b \text{ ATE } d) & (c \text{ ATE } d))\]

*b and c ate* d *iff* b ate d and c ate d

Or:

\[(i \text{ SAW } b) \equiv ((i \text{ SAW } b) & (i \text{ SAW } c))\]

*I saw* b and c *iff* I saw b and I saw c

It very important to keep in mind that sometimes 'and' does not correspond to the period. For example, in 'Chicago is between San Francisco and New York' it does not follow that 'Chicago is between San Francisco and Chicago is between New York'.


The ⊢ symbol has basically the same meaning as it does in other logics where \( b \vdash c \) usually means \( c \) is provable from \( b \). First, as the reader may have noticed, we do not like the word 'prove' because it is too strong a word, we prefer the word 'argue'. Second, 'arguing that \( c \) follows from \( b \)' has a very precise meaning in this system:

54. \( b \) and \( c \) entails a consistent set of sentences.
55. \( b \) and not \( c \) entails a contradictory set of sentences.
56. not \( b \) and \( c \) entails a consistent set of sentences.
57. not \( b \) and not \( c \) entails a consistent set of sentences.

Second, the difference between \( \vdash \) and \( \rightarrow \) is that the latter is only used in definitions and axioms. For example, suppose you have two atomic sentences connected by \( \rightarrow \). You cannot show that the affirmation of \( p \) in conjunction with the denial of \( q \) leads to a contradiction because atoms are not equivalent to anything. For this reason, \( \rightarrow \) is a lot like a promise. What we are saying when we assert \( p \rightarrow q \) is that we are never going to believe \( p \) and \(~q\) at the same time, hence whenever we believe \( p \) we are going to infer \( q \). The \( \vdash \) sign is not a promise but is a demonstration that we are actually keeping our promises. Because the \( \vdash \) sign connects molecular statements, it can be shown that the affirmation of \( p \) and the denial of \( q \), if connected by \( \vdash \), does in fact lead to a contradiction.

The \( \iff \) is the same except that in 56 'consistent' is replaced with 'contradictory'.

The ⊳ symbols stands for the exclusive disjunction and we will explain in the statement logic why we use it so much more often than the ∨ symbol.

The # symbol is my own invention and we will talk about it in more depth in the statement logic section, but for now we should at least say a few things about it. The # connects two unrelated sentences, that is to say, \( p \# q \) means that \( p \) and \( q \) are consistent, \( p \) and \(~q\) are consistent, \(~p\) and \( q \) are consistent and \(~p\) and \(~q\) are consistent. We will save the other symbols for when we discuss statement logic.

That sums up our discussion of the atoms. In closing we should state that it is very unlikely that these atoms are exhaustive. Although the pace of atomic discovery has slowed down considerably, I'm quite confident that there are more atoms remaining to be discovered. Logicians reading this paper are encouraged to act now and pick the low-hanging fruit while it is still there. We here list a second table of the atomic non-literals:

### 3.13. Table of Atomic Relations

<table>
<thead>
<tr>
<th>Relations</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>above</td>
<td>AB</td>
<td>relates points</td>
</tr>
<tr>
<td>after</td>
<td>A</td>
<td>relates moments</td>
</tr>
<tr>
<td>after¹</td>
<td>AL</td>
<td>relates letters</td>
</tr>
<tr>
<td>after²</td>
<td>AR</td>
<td>relates the relata and relations in a relationship</td>
</tr>
<tr>
<td>after³</td>
<td>G</td>
<td>relates numbers, e.g. 4 is after 3</td>
</tr>
<tr>
<td>are³</td>
<td>J</td>
<td>relates a thing to a property in adjective form</td>
</tr>
</tbody>
</table>
are\$ \quad I \quad \text{relates an instance to a universal}

at \quad S \quad \text{relates a void or a particle to a point}

at\w \quad P \quad \text{relates a thing to a possible world}

at\i \quad M \quad \text{relates a relationship to an imagination}

at\o \quad N \quad \text{relates a whole or set to a number}

at\u \quad O \quad \text{relates a sensation to a point in the sensorium}

at\t \quad T \quad \text{relates a thing to a moment}

at\y \quad Z \quad \text{relates a thing to a two-dimensional point}

believe \quad B \quad \text{relates a mind to a belief}

desire \quad D \quad \text{relates a mind to a desire}

has\w \quad W \quad \text{relates a whole to a part, or a group to a member}

have \quad H \quad \text{relates a thing to a property in noun form}

in front of \quad F \quad \text{relates points}

is\v \quad V \quad \text{relates a thing to a property in adverb form}

is\y \quad Y \quad \text{relates a thing to a property which is equivalent to a determiner}

left of \quad L \quad \text{relates points}

in\u \quad U \quad \text{relates a possible world to a possible universe}

in\l \quad IL \quad \text{relates a relationship to a language}

think about \quad C \quad \text{relates a mind to a thing}

In the following table, the sentences which begin with e.g. are not exact renditions of the logical notation but are more colloquial sentences which can in theory be derived from the notation.

### 3.14. Table of Atomic Non-Relations

- **real**
  \[(c=\text{real}) \rightarrow ((b \ J \ c) \ & \ (d \ P \ b))\]
  e.g. If \(c\) is an abbreviation of 'real' then \(c\) is the property of a possible world.

- **extant**
  \[(c=\text{extant}) \rightarrow (b \ J \ c)\]
  e.g. If \(c\) is an abbreviation of 'extant' then \(c\) is a property.

- **now**
  \[(c=\text{now}) \rightarrow (b \ T \ c)\]
  e.g. If \(c\) is an abbreviation of 'now' then \(c\) is a moment.

- **here**
  \[(c=\text{here}) \rightarrow (b \ S \ c)\]
  e.g. If \(c\) is an abbreviation of 'here' then \(c\) is a point.

- **consistent**
  \[(c=\text{consistent}) \rightarrow ((p \ J \ c) \ & \ (p \ I \ d)) \ & \ (d=\text{relationship})\]
  e.g. If 'consistent' is a property of \(p\), then \(p\) is a relationship.

- **energy**
  \[((b \ H \ c) \ & \ (c \ I \ d)) \rightarrow (b \ S \ e)\]
  e.g. If \(b\) has [some] energy then \(b\) [exists] at [a] point [in space]. The converse does not hold since there could be a void of space with no energy in it.
many & (c=many) & ((b J c) → (b W d)) & (((b J c) & (d ~ J c) & (b N e) & (d N f)) → (e G f))
e.g. If b is many, then b is a group. And if b is many and d is not many then b [exists] atₙ e [in numerical space] and d [exists] atₙ f [in numerical space] and e [is] greater than f.

3.15. Table of Atomic Non-Literals

& (undefined)
⇒ (b⇒c) means wherever we see b we may replace it with c but not vice versa
⇔ If the symbols to the left or the right of ⇔ are written on a line, then we may write the symbols on the other side of ⇔ on a different line.
⇨ If the symbols to the left of ⇨ are written on a line, then we may write the symbol to the right of ⇨ on a different line.
⇿ The symbol to the left of ⇔ is an abbreviation of the symbols on the right which is a relationship or a set of relations
= ([abbreviation]=[word]), the word on the right may be replaced with the abbreviation on the left.
¬ p & ¬p are consistent
¬ p & ¬p are not consistent
≈ If we have (b≍c) on line 3 and SUB 2,3 in the justification section then only in line 2 may we replace b with c but not vice versa.

3.16. Wild Disjunctions

According to Turchin, it was the great David Hilbert himself who coined the term metalanguage, though he may have only used the word 'metamathematics'. But the historian of linguistics, Koerner, has only been able to find its earlier usage in Carnap's 1942 Introduction to Semantics. We frankly do not think this idea applies to natural language. There is only one language and one set of indefinable words, not a metalanguage and an object language. It might turn out that we cannot eliminate metalanguage from our system but so far it seems that we are succeeding. In any case, there are certain words which have a very distinctive metalinguistic feel to them and which share the common property of being defined disjunctively. Wild disjunction, it should be noted comes from Fodor, not because we think he was making a good point, but because it sounds so funny, who wrote: "banal considerations suggest that a physical
description which covers all such events must be wildly disjunctive." It is very important that we eliminate this distinction between metalanguage and object language because all too often logicians come up with jargon which they claim is metalinguistic and turns out to be just as mysterious as the object language which they are trying to clarify.

So the definition of a noun is just a long disjunction, "b is a noun iff b is a dog or b is a cat or b is a house or b is a table" etc. The definition of an adjective is nothing other than "b is an adjective iff b is red, or b is tall, or b is large, or b is smart" etc. In short, these wild disjunctions are simply lists. So we define abbreviation by simply making a list of the allowable abbreviations which are b, c, d, e, etc. Admittedly, to assert that some words are definable is somewhat of a stretch. So the definition of variable is:

\[(b \equiv \text{variable}) \equiv (d \equiv (b \Rightarrow d)) \& (c=\text{variable}) \& (e=\text{consistent})\]

\[b \text{ is a variable iff it is consistent that b is instantiated by something.}\]

It's not at all clear if that definition works. But it seems to so we're sticking with it. Probably the other stereotypical metalinguistic word which is hardest to define is substitution. In our system, it is defined as:

\[(b \text{ can be substituted with } c \text{ and vice versa}) \equiv (b = c)\]

If it turns out that I can persuade very few deductive metaphysicians that this is an acceptable definition of substitution then I will abandon this belief.

### 3.17. Prepositional Relations

In this system a standard sentence can be composed of a high number of relations. So "I saw Leibniz in the house" eventually reduces to:

\[(i \text{ SAW b INM c) & (c I d) & (b=leibniz) & (d=house)}\]

In other words, we put the relations SAW and INM in the same sentence. If we wanted to add in time then we would just write:

\[(i \text{ SAW b INM c T f) & (c I d) & (b=leibniz) & (d=house) & (f=4 o'clock)}\]

*I saw Leibniz in the house at four o'clock.*

If we want to express a fact about a possible world such as 'Hillary won the election in possible world b in 2017', this would be expressed as:

\[(d \text{ WON c P b INB e) & (c I f) & (d=hillary) & (f=election) & (e=2017)}\]

We have to use the INB relation rather than the INM relation because the object of the INM relation is always a set of particles whereas the object of the INB relation is always a period of time.
4. A Few Axioms and Lemmas

4.1. Ontology

Every word in this system needs to belong to a category. A large number of nonsensical sentences are nonsense precisely because we assert that a member of one category has the properties of a member from a mutually exclusive category. For example, 'the point thought', 'the moment ran', 'the property smelled', 'the concept slept', all these are falsified precisely because things which think, run, smell or sleep belong to a category which point, moment, property and concept do not belong to. For those words which are basic or almost basic we cannot infer what category they belong to through their definition but must simply assert their category just as we simple assert definitions. We are not going to use trees to express our categories because it takes up too much space and also because our categories change so much that it is too laborious to keep redrawing the trees. Instead we specify the supercategory by being on the left side of the \( \equiv \) sign and then on the right side we specify the subcategories by being divided by the \( \vee \) sign which stands for exclusive disjunction:

58. \( \text{thing} \equiv \text{universal} \vee \text{particular} \)
59. \( \text{universal} \equiv \text{relation} \vee \text{class-concept} \vee \text{property} \)
60. \( \text{property} \equiv \text{noun property} \vee \text{adjective property} \vee \text{determinative property} \vee \text{adverbial property} \vee \text{relationship property} \)
61. \( \text{determinative property} \equiv \text{number} \vee \text{other} \)
62. \( \text{particular} \equiv \text{relationship} \vee \text{non-relationship} \)
63. \( \text{relationship} \equiv \text{fact} \vee \text{falsehood} \vee \text{fiction} \)
64. \( \text{non-relationship} \equiv \text{whole} \vee \text{non-whole} \)
65. \( \text{non-whole} \equiv \text{letter} \vee \text{symbol} \vee \text{mind} \vee \text{moment} \vee \text{particle} \vee \text{void} \vee \text{point} \vee \text{God} \vee \text{instantiated property} \vee \text{instantiated relation} \vee \text{sensation} \)
66. \( \text{whole} \equiv \text{arbitrary whole} \vee \text{sensorium} \vee \text{possible world} \vee \text{imagination} \vee \text{spatial whole} \vee \text{temporal whole} \vee \text{material whole} \vee \text{symbolic whole} \)
67. \( \text{temporal whole} \equiv \text{unique event} \vee \text{process} \)
68. \( \text{possible world} \equiv \text{reality} \vee \text{unreality} \)
69. \( \text{universal} \rightarrow \text{non-whole} \)
70. \( \text{relationship} \rightarrow \text{whole} \)

It also helps to list the synonyms of the categories:
We have not yet been able to fit 'part' or 'member' into our ontology, as well 'instance'. Perhaps this is because just about everything except for the universe is part of something else. We also could not fit 'wholes' or 'groups' neatly into our ontology so we simply specify that universals are non-wholes in the sense that they do not have parts and that relationships are wholes in the sense that their relata and relations compose them. It should also be pointed out that 'instance' is not synonymous with 'particular' since universals can be instances of other universals but particulars cannot ever be universals. For example, 'rock' is a universal which is an instance of 'physical universal'.

I think also most of these words are understandable with the exception of 'instantiated property' which we take to be synonymous with trope. Just as computer programmers are not very good at naming their rules, so too philosophers are not very good at naming the concepts they claim to have discovered. Finding the right name for a discovered concept can sometimes determine whether or not someone else is persuaded of its real existence. had the trope theorists named their discoveries what they are, namely, instantiated properties, I would have believed in them a year earlier. I'm sure other philosophers deny their real existence precisely because they have been poorly named.

Briefly put, if you want to say that 'this redness is different than that redness' then in order to avoid contradiction you have to believe that the property redness has instances which are different. That is all tropes or instantiated properties are. We do not believe, however, that instantiated properties are the "very elements of being" as Donald Williams puts it.

4.2. The Lemma of Entity

The lemma of entity simply asserts the enormously trite belief that 'everything is a thing'. And in this system when we say 'thing' we mean literally everything is a thing and that includes numbers, grammatical particles, letters, minds, contradictions, fictional characters, God, etc. So when Kant says "we therefore assert the empirical reality of space (with respect to all possible outer experience), though to be sure at the same time its transcendental ideality, i.e., that it is nothing," we wholeheartedly disagree. In early versions of our system we thought that 'thing' was indefinable but now we believe that it
is defined as 'either a universal or a particular' and we can then argue that everything in our ontology is a thing since our ontology is meant to literally fit everything into a class. Currently, the lemma of entity is hard-coded into our system, but eventually we would like to show how it is argued that certain objects are instances of the category 'thing'. The lemma of entity is spelled in our formal arguments as LE ENT.

4.3. The Ontological Lemmas

Take the following conjunction:

\((b \land c) \land (d \land b)\)

This entails a contradiction in our system. The subject of \(b\) must always be a mind and the object of \(S\) must always be a point. And we see from our ontology on line 65 that points and minds are subsets of non-wholes which means that they belong to mutually exclusive categories. Currently, we have simply hard-coded this into our machine that the following is a lemma:

\(((b \land c) \land (e \land f)) \vdash (e \sim S b) \land (f=\text{thing})\)

We then instantiate \(e\) with \(d\) and get:

\(((b \land c) \land (d \land f)) \vdash (d \sim S b)\)

We then use the lemma of entity to infer that \(d\) is a thing, like so:

\((d \land f)\)

We then use conjunction elimination and introduction so that we may write:

\((b \land c) \land (d \land f)\)

We then use modus ponens to infer:

\((d \sim S b)\)

And we thus infer the contradiction:

\((d \sim S b) \land (d S b)\)

There are four possible ways two atomic relationships can be related:

The subject of each can be the same, type SS:
(b B c) & (b S d)

The subject of the first can be the same as the object of the second, type SO:

(b B c) & (d S b)

The object of the first can be the same as the subject of the second, type OS:

(b B c) & (d S c)

And finally, the object of the second, can be the same as the object of the second, type OO:

(b B c) & (d S c)

If you do the math, since there are 24 atomic relations and four positions, that means we have to specify which of the $24 \times 24 \times 4 = 2304$ different combinations are consistent or contradictory. It turns out that it only requires about 50 lines of code and it is very easy to do.

So we name each of these lemmas as follows. We start with LE then place a period, then the first relation, then the next relation then the type. So the above lemma would be called:

LE.B.S.SO

Again, we would eventually like to argue for all of these lemmas but for now they are simply hard-coded.

4.4. The Axioms of Identity

There is nothing in the definition of 'same' that states that two different events cannot occur at the same moment, or two different particles cannot occupy the same point. In fact, 'same' is not even defined in this language. You simply have to know that b and b are the same abbreviation. For this reason, we simply need to state the rules which govern the use of the word 'same' and 'different'.

We first need to dispense with the myth of numerical identity. It is often asserted that the only thing which is identical is that which is identical to itself. It might even be a more certain analytic truth than the Law of Non-contradiction since the LNC now has
opponents. But how are we to even formalize this belief? Suppose we were to state something along the lines of:

71. b is numerically identical to b.

So do the two b's refer to the same thing? They do not. The first b refers to a thing which has the property 'was referred to at time 1 with the letter b and the second b has the property was referred to at time 2 with the letter b. Let's try once again to find a consistent sentence which uses the word 'numerically identical':

72. Brad and Angelina starred in a numerically identical film.
73. Brad and Angelina starred in b and b is a film.

There is no difference in meaning between 72 and 73. The term 'numerically identical' in 72 serves absolutely no purpose and is simply redundant. Let's try once again to find a consistent, non-redundant use of the word 'numerical identical':

74. Brad and Angelina starred in Mr and Mrs Smith and Mr and Mrs Smith is numerically identical with itself.

If we abbreviate Mr and Mrs Smith with b and make b the object of the relation 'is numerical identical with' we are back to the same problem we had in 71. How about symbols? Certainly they can be the same, can't they? No. Observe:

75. b is the same as b.

The first b has the property 'being written at time 1' and the second b has the property 'being written at time 2'. Other attempts to salvage 'numerical identity' are even worse than 71:

76. Mark Twain is numerically identical to Samuel Clemens.

76 fails because there is someone out there who believes that Samuel Clemens is different from Mark Twain. So we have two objects one which has the property 'thought by b to be different from Mark Twain' and another object which does not have this property. When we try to revise 71 to expel the contradiction, we begin the process of establishing an arbitrary distinction between 'having so many different properties that it should be referred to with a different abbreviation' and 'not having a sufficient number of different properties such that it requires a different abbreviation'. In short, quite literally everything is different we just have to establish our rules ahead of time as to what is so different that it must be referred to by what we recognize as a different abbreviation.

In short, we have to go through all of our atomic relations and decide whether or not the same abbreviation can exist in two different positions. For instance, take the following:

77. (b N c P d T e) & (f N c P d T e)
$b$ instantiates [number] $c$ at $P$, $d$ at $T$, $e$ and $f$ instantiates [number] $c$ at $P$, $d$ at $T$, $e$

78. (b N c P d T e) & (b N f P d T e)
$b$ instantiates [number] $c$ at $P$, $d$ at $T$, $e$ and $b$ instantiates [number] $f$ at $P$, $d$ at $T$, $e$

No category error is committed in 77 and 78. The only abbreviations which exist in the same position of an atomic relation in 77 are c, d and e and they exist in the same position in both sentences. In 78 it is b, d and e which exist in the same position of the atomic relations. 78 is false whereas 77 is true. This is because we are asserting that the same set instantiates two different numbers at the same time in the same possible world. There is nothing in the definition of 'number' or 'same' that falsifies 78 so we have to make it axiomatic. In the vast majority of cases the same object can exist in the same relational position in two different sentences. There are five exceptions to this rule:

79. ((b S c P g T f) & (d I e)) → (d ~ S c P g T f) & (e=particle \lor void)
  e.g. Two different particles cannot exist at the same point at the same time in the same possible world.

80. ((b S c P g T f) & (d I e)) → (b ~ S d P g T f) & (e=point)
  e.g. The same particle cannot exist at different points in space at the same time in the same possible world.

81. ((b N c P g T f) & (d I e)) → (b ~ N d P g T f) & (d=integer)
  e.g. The same object cannot be counted by different numbers at the same time in the same possible world.

82. (((b P c T d) & (e I f)) → (b ~ P c T e)) & (e=moment)
  e.g. The same possible world cannot exist at different times.

83. ((b O c) & (d I e) & (f I g)) → ((d ~ O c) & (b ~ O f))
  e.g. The same sensation cannot exist in different sensoriums and the same sensational point cannot be occupied by different sensations.

I realize that scientists believe that different bosons can occupy the same point in space but it has to be that this is some sort of logical error. In any case, it does not really matter at this point since we almost never discuss the nitty-gritty of particle physics. I think few people doubt 80 and 81. 83 concerns sensations and it will be a very long time before we begin to research the mind, so if 83 is wrong, we have plenty of time to change it.

82, however, is controversial. We have to make it perfectly clear that these axioms were chosen because they make things easy. Recall that everything is different, it's just that we have to lump things together into groups and refer to everything within that group with the same symbol, otherwise communication would be possible. For this reason we can literally adopt any set of axioms of identity we want, it's just that these axioms turned out to be the easiest to calculate with. So we believe that the same person can exist at
different moments but the same possible world cannot exist at different moments. Why is that? That seems inconsistent, doesn't it? It turns out that the calculations are much easier if we use that axiom. So as to have a reasonable theory of probability we need a system where the same possible world cannot exist at different moments. Further, we could easily have a system where the same person cannot exist at different moments, but it turns out that trying to formalize motion in sentences such as 'Bertrand moved' turn out to be very difficult if we make it a rule that the same object cannot exist at different times.

Another controversial move is an axiom that is not on the list, which would be:

84. ((b B c) → (c ~ P d)) &(f=possible world)

What this would mean is that there is a strict separation between beliefs and facts. So no fact would exist in your mind, only beliefs which may or may not correspond to facts. So what we say in 84 is that any belief cannot exist in a possible world but only in your imagination. We tried building a system with that as an axiom but it turned out to be too complicated. Any philosopher who can build a simple and easy to use system with that as an axiom, I will be happy to adopt it.
5. The Instantiation Rules

5.1. Why We do not Use $\forall$ and $\exists$

All redundant symbols must be deleted. If it can be shown that $\forall$ and $\exists$ are redundant then we would be contradicting ourselves if we continued to use them. Further, they look ugly and are very difficult to program with. As for the charge that they are ugly we can easily solve that problem by getting rid of scope and just marking each universally quantified variable with a prime sign and each existentially quantified variable with a double prime sign. But do we even need the prime signs? We do not. First, the quantifiers are only used in hypothetical sentences. So our first step in determining whether or not an abbreviation is universal, existential or constant is to simply recognize that if a variable only appears in a hypothetical sentence then it is either universal or existential. Our second step is to ask ourselves if we really understand what we mean when we use the words 'universal' or 'existential'. The only distinction that I have been able to make is that between those abbreviations which exist on both sides of a hypothetical connective and those that do not. We will call the former 'full' and the latter 'half'. For example, in "if a person $b$ owns a $c$ and $c$ is a car, then $b$ is a $d$ and $d$ is the concept car-owner," $b$ exists on both sides of the $\rightarrow$ connective whereas $c$ does not.

Why is this distinction important? It is only because I expose myself to contradiction if I assert something twice about $b$, once about $c$ and once about $d$. For example, in 'Walt is a person, he owns a car and he is not a car-owner' I have just referred twice to the same person and referred once to cars and once to car-owners and I have contradicted myself. But in 'Milton is a person, this Ford is a car and someone own this Ford', I have only referred once to the same person, but twice to the same car and I have also referred to a car-owner but not stated their identity. I have not contradicted myself precisely because it is the abbreviation $b$ which must be referred to twice in order to obtain a contradiction and hence this abbreviation has properties slightly different from the properties of $c$ or $d$.

It is for this reason that we can get rid of $\forall$ and $\exists$. We didn't really understand what we were talking about when we used the words 'universal' and 'existential'. We just gave them fancy names, said some vague things as to what they meant and never really asked ourselves if we had a clear rule that distinguished them. There is one potential counterexample:

85. For some $b$, if $b$ is a mammal then $b$ is a cat.
This is not a natural language sentence, but something a logician made up. What they really mean to express is:

86. If b is a mammal then it is possible that b is a cat.
87. If b is a group of mammals then some of them are cats.

86 will be formalized in the near future when we come to the necessary/contingent distinction, and 87 is formalized in the very distant future in the section on 'some'.

5.2. The Rules Proper

All of our interesting inferences are done with instantiation since the statement logic is usually nothing more than modus ponens and ≡ elimination. The instantiation rules are not very difficult to use or understand when calculating by hand but this is because calculating by hand is slow which prohibits one from encountering a high number of bizarre situations. Once one sees all the bewildering possibilities which computer calculation makes possible one realizes how mind-numbingly difficult these things actually are. To give the reader an idea of how difficult this problem is, bear in mind that for even rather straightforward sentences of roughly 10 words, we will still use roughly 20 to 40 abbreviations. One then has to go through each pair of abbreviations and determine whether or not one can be instantiated for the other. So for 20 abbreviations, one can form a matrix like so:

```
  b  c  d  e  f  g  h  j  k  l  m  n  o  p  q  r  s  t  u  v
b x x x x x x x x x x x x x x x x x x
# # # # # # # # # # # # # # # # # #
  c x x x x x x x x x x x x x x x x x x
  d x x x x x x x x x x x x x x x x x x
  e x x x x x x x x x x x x x x x x x x
  f x x x x x x x x x x x x x x x x x x
  g x x x x x x x x x x x x x x x x x x
  h x x x x x x x x x x x x x x x x x x
  j x x x x x x x x x x x x x x x x x x
  k x x x x x x x x x x x x x x x x x x
  l x x x x x x x x x x x x x x x x x x
  m x x x x x x x x x x x x x x x x x x
  n x x x x x x x x x x x x x x x x x x
  o x x x x x x x x x x x x x x x x x x
  p x x x x x x x x x x x x x x x x x x
  q x x x x x x x x x x x x x x x x x x
  r x x x x x x x x x x x x x x x x x x
  s x x x x x x x x x x x x x x x x x x
  t x x
  u x
```
One has to check whether or not a pair of abbreviations can be instantiated one for the other in each box with an x marked in it. That's 190 pairs. To make matters worse, the properties of the abbreviations change as one makes inferences. Now imagine doing that for 2000 sentences. Fortunately, there are relevance rules one can use to filter out pairs which one knows in advance cannot result in an instantiation. The problem with these relevance rules is they often only working 99% of the time. So if one relevance rule manages to filter out 75% of all pairs 99% of the time then one has to abandon the relevance rule for a weaker one. Further, the correct programming of the relevance rules often uses more time than one gains had one not used them in the first place.

**Definition 1: Full General Variable**

*b* is a full general variable iff

1) *b* appears in any hypothetical sentence in our dictionary which is not a constant and appears on both sides of the $\equiv$, $\rightarrow$, $\lor$, $\forall$ or $\#$ connectives or,

2) *b* appears in more than one sentence of a negated conjunction of the following form $\neg(b \& c)$.

**Definition 2: Half General Variable**

*b* is a half general variable iff

1) *b* appears in any hypothetical sentence in our dictionary which is not a constant and appears on one side of the $\equiv$ connective or,

2) *b* appears in the antecedent of the $\rightarrow$ connective but not the consequent, or,

3) *b* appears in only one sentence whose main connective is $\lor$ or $\forall$ or,

4) *b* appears in only one sentence of a negated conjunction.

For example, in the definition for particle, *b* is a full general variable, *c* and *f* are constants and *d* and *e* are half general variables:

88. (c=particle) & ((b I c) $\equiv$ ((b $\lor$ d) & (b $\lor$ e) & (e I f))) & (f=energy)

The reason why we make this distinction is because only full general variables enable us to find contradictions. This is quite hard to explain right now, but after all the rules are stated we will return to this point and show why it is true.

**Definition 3: Indefinite Abbreviation**

*b* is an indefinite abbreviation iff

1) *b* is not a constant and

2) it is not explicitly stated that *b* has the property 'being definite' and one of the following two conditions is met:
2a) *b* appears in the consequent of a conditional or,
2b) *b* appears in a detached sentence.

**Definition 4: Relational Property**

*b* is a relational property of *c* iff *b* is composed of the standard sentence that *c* appears in but *c* is replaced with *α*.

For instance, in the sentence (*b B c*), *bBα* is a relational property of *c*, and *αBc* is a relation property of *b*. If the sentence contains a prepositional relation, such as (*i SEE b IN c*), then its relational property would be (*iSEEαINc*). If the sentence contains a prepositional relation and one of the other abbreviations is a general variable then this variable is replaced with *γ* since we think *β* looks too much like a capital letter. And *γ* can be instantiated by any abbreviation. So if we are trying to determine the property of *b* in the sentence (*i SEE b IN c*) where *c* is a general variable then it would be formalized as (*i SEE α IN γ*).

**Definition 5: Lone Property**

*b* is a lone property of *c* iff *c* is a general variable and the other abbreviation to which it has a relation is also a general variable.

In this case, its property is simply 'being the subject of the R relation', whatever that relation may be and is written as *αR*.

**Definition 6: Definite Relation**

*b* is a definite relation iff *b* is composed of a relation and a constant.

Usually, this relation is J or I followed by the object which is almost always a constant. If no constants appear in the sentence then the definite relation is simply the relation itself.

**Definition 7: Property Set**

*b* and *c* are properties which belongs to set *d* iff *b* and *c* are properties of general variables which belong to the same conjunction embedded in a hypothetical sentence.

For example, in the definition of 'between' whose relata are numbers:


text

*b* has three property sets, but two of them are the same. In the biconditional antecedent there is no conjunction so *b* only has one lone property which is *αBTW* since *c* and *d* are general variables too. In the conjunction within the first disjunct, it has two lone properties *αG* and *Gα*, which are the same properties it has in the second disjunct.
To take a more fictional example, so as to convey the pattern:

\[
89. (((b \ A \ c) & (b \ B \ d)) \equiv (((b \ C \ e) & (e \ D \ b)) \lor ((b \ E \ f) & (b \ C \ g)) \lor ((b \ D \ h) & (b \ H \ k) & (b \ I \ m))) & (m=\text{word})
\]

In 89 b has 4 property sets. In the biconditional antecedent its properties are: \(\alpha A, \alpha B\). In the first disjunct of the biconditional consequent, its properties are \(\alpha C, \alpha D\). In the second disjunct, they are: \(\alpha E, \alpha C\). In the third disjunct, they are: \(\alpha D, \alpha H, \alpha I m\)

**Instantiation Rule 1**

*Abbreviations in hypothetical sentences must first be translated into new abbreviations such that two different abbreviations do not refer to the same thing.*

For example, as we mentioned earlier the lemmas for the INM relation and 'moment' are as follows:

\[
((bIc) \rightarrow (dAb)) & ((bIc) \rightarrow (bAe)) & (c=\text{moment})
\]

\[
(b\text{INMc}) \rightarrow (((bWd) \equiv ((dSe) & (cWe))) & (bWf) & (fSg) & (hWg))
\]

We can leave the first lemma as it is, but in the second lemma we must translate b, c, d, and e into different abbreviations since those abbreviations already refer to something different in the first lemma. We would first list our translations as follows:

(b\equiv j) (c\equiv k) (d\equiv m) (e\equiv n)

Then in the second lemma wherever we see b, for example, we would replace it with j:

\[
(j\text{INMk}) \rightarrow (((jWm) \equiv ((mSn) & (kWn))) & (jWf) & (fSg) & (hWg))
\]

**Instantiation Rule 2**

*If a constant appears in a hypothetical sentence and that constant has already been abbreviated then each time that constant appears it must be given the same abbreviation.*

For example, we referred to particle \(^n\) and energy in 88 and abbreviated them as c and f respectively. If we should refer to particle \(^n\) and energy again in another hypothetical sentence then we must use c and f.

**Instantiation Rule 3**

*If a detached abbreviation has all the properties that a full general variable has in any one of its property sets, then we may substitute that general variable for the detached abbreviation so long as none of the other instantiation rules are broken.*
For example, suppose abbreviation $b$ has three property sets. In one set its property are $c$, $d$, and $e$. In the second set, the properties are $f$, $g$ and $h$. And in the third set its properties are $j$, $k$, and $m$. Now, suppose the detached abbreviation, $n$, has properties $p$, $q$, $f$ and $h$. If this is the case, since it has all of the properties that $b$ has in set two, then we may make an instantiation provided no other rule is broken. Now suppose that the detached abbreviation $o$ has properties $f$, $d$, and $m$. Even though all of those properties are properties of $b$, $o$ does not have all of the properties found in any one set and hence no instantiation can be made.

**Instantiation Rule 4**

*If a general variable has a lone property, then a detached abbreviation has that property if every abbreviation or relation or negation sign which appears in the lone property also appears in one of the detached abbreviation's properties.*

For example, if the lone property $\alpha F$ and the detached property is $\alpha Fc$, then an instantiation may occur if that is the only property that the general variable has in one of its property sets.

**Instantiation Rule 5**

*If a full general variable $b$ has a relation to a half general variable $c$ then the detached abbreviation which has the same relational property as $b$ must also have a relation to another detached abbreviation which has all the relational properties that $c$ has.*

**Instantiation Rule 6**

*If a sentence is detached from a hypothetical sentence which contains an abbreviation which was once modified by 'a$^{h}$' then each time it is detached the indefinite abbreviation must be translated into a new abbreviation.* (See section 6.4.6. for an explanation of a$^{h}$)

For example, from

If $b$ plays in the orchestra, then $b$ plays an$^{h}$ instrument.
Will plays in the orchestra.
Kate plays in the orchestra.

it does not follow that Will and Kate play the same instrument. The above argument would eventually be rendered in standard form as:

$$(\text{e=instrument}) \land (\text{f=orchestra}) \land (\text{g=definite}) \land (\text{h=will}) \land (\text{j=kate})$$

$$(b \text{ PLI } c) \land (c \text{ I f}) \land (c \text{ J g}) \rightarrow ((b \text{ PL } d) \land (d \text{ I e}))$$

$$(j \text{ PLI } c) \land (c \text{ I f}) \land (c \text{ J g})$$

$$(h \text{ PLI } c) \land (c \text{ I f}) \land (c \text{ J g})$$
When we instantiate b with j we must translate d into a new abbreviation and when we instantiate b with h we must translate d into an abbreviation different from the one that we used when we instantiated b with h. Otherwise, it would follow that Will and Kate would play the same instrument which is obviously not the case. Since d is an indefinite abbreviation, it follows that it is contingent that they play the same instrument, even though it is rare in orchestras that members share instruments.

**Instantiation Rule 7, The Axiom of Difference**

*In a hypothetical sentence all abbreviations are considered to be different and hence none can be instantiated for the other, nor can indefinite identity be used on them unless explicitly stated otherwise.*

We call this the Axiom of Difference. We will discuss indefinite identity in section 6.3.11. For example, in: $((b = c) \not\equiv (d < b)) \land ((b = c) \not\equiv (b < a))$ & ($c = \text{moment}$), b, c, d, and e are considered different and hence cannot be instantiated or made identical to each other through indefinite identity.

As simple as those rules appear it has taken us three years and probably 30 alterations of those rules to come up with them and we're quite certain that more alterations are to come in the future since the pace of alteration still has not slowed down yet. Let's now do some examples. Suppose our hypothetical sentence is:

$((c = \text{particle}^n) \land ((b = c) \equiv ((b = S d) \land (b = H e) \land (e = f))) \land (f = \text{energy}))$

And our detached sentences are:

$\text{(g S h)}$
$\text{(g H j)}$
$\text{(j I f)}$

In our hypothetical sentence d simply has one relational property which is 'being the object of the S relation' which h also has. e, however, has two properties, it is the object of the H relation and it is an instance of f which is a constant. j has both of those properties, hence g has all the properties that b has. We then write:

$(b \Rightarrow g) \land (e \Rightarrow j) \land (d \Rightarrow h)$

We then make our replacements like so:

$((g = c) \equiv ((g = S h) \land (g = H j) \land (j = f)))$

We then use conjunction introduction and get:

$((g = S h) \land (g = H j) \land (j = f))$

We now use the rule $\equiv E$ and we can infer:
Now, suppose our detached sentences were:

\[(g \ I \ c)\]

\[(g \ S \ h)\]
\[(g \ H \ j)\]
\[(k \ W \ j)\equiv((m \ TK \ j) \ & \ (j \ I \ n))\]
\[(n=\text{void})\]
\[(m=\text{Leibniz})\]

In this case, \(j\) does not have the property 'being an instance of energy' but has the property 'being thought about by Leibniz', or more precisely, is a member of the group 'voids thought about by Leibniz'. Hence, \(b\) cannot be replaced with \(g\) since it does not have all the relational properties that \(b\) has on the biconditional consequent. Now suppose our detached sentences were:

\[(g \ I \ c)\]
\[(g \ S \ h)\]
\[(g \ H \ j)\]
\[(k \ W \ j)\equiv((m \ TK \ j) \ & \ (j \ I \ n))\]
\[(n=\text{void})\]
\[(m=\text{Leibniz})\]

We now must replace \(b\) with \(g\) because \(g\) has the property 'being a particle', \((b \ I \ c)\) which is the only property had by \(b\) in the antecedent biconditional. This will result in a contradiction because as we saw on line 279 of our ontology, voids and particles belong to mutually exclusive categories. Even though this leads to a contradiction we must make the substitution because that is exactly what the rules are and this is practically the only way we get contradictions in our system. Let's do another example. Suppose our hypothetical sentence is:

\[(c=\text{proper name}) \ & \ ((b \ I \ c)\equiv((b \ I \ d) \ & \ (b \ \text{RF} \ e) \ & \ (e \ I \ f))) \ & \ (d=\text{word}) \ & \ (f=\text{individual}) \ & \ (\text{refer}=\text{RF})\]

\(b\) is\(a\) proper name iff \(b\) is\(a\) word and \(b\) refers to \(e\) and \(e\) is\(a\) [an] individual.

And our detached sentences are:

\[(g \ I \ d)\]
\[(g \ \text{RF} \ h)\]
\[(h \ I \ f)\]
\[(h=\text{kiera knightley})\]

In this case, \(b\) is a full general variable, \(c\), \(d\) and \(f\) are constants and \(e\) is a half general variable. In our detached abbreviations, \(g\) has all the properties that \(b\) has on the right side of the biconditional and \(h\) has all the properties that the half general variable \(e\) has
since e has only two properties: it is the object of the 'refer' relation and is an instance of the universal 'word'. h also has the property 'being an abbreviation for keira knightley' but that does not prohibit it from being instantiated by e because it need only have all the properties that e has. If it has additional properties then that does not matter.

5.3. Why Only Full General Variables Help us Obtain Contradictions

It should now be clear why full general variables succeed in obtaining contradictions and half general variables do not. For example, suppose your hypothetical sentence is:

$$(((b \land c) \land (b \land d)) \rightarrow (b \land e)) \land (e \land f) \land (f=\text{definite})$$

If you find an abbreviation that matches the property of c or the property of d, then it will not help you obtain a contradiction because these abbreviations have to have the same negation value as their detached counterparts in order to be instantiated. That is not the case with b however. So suppose your detached sentences were:

$$(m \land n) \land (m \land o) \land (m \sim A e)$$

It is not the case that for each property that b has m must also have that property and the same negation value. They can differ in their negations values, just as m does not have the A relation to e whereas b does. c and d, on the other hand, must share all properties with n and o, including the negation sign and hence c and d will never succeed in enabling us to infer a contradiction.

5.4. Relevance Rules and Decision Procedure for Instantiation

We cannot define every object and check whether or not it can be instantiated by any other object in the argument. This can slow an argument down by a factor of 3 or even a factor of 10. Speed is of the utmost importance. A software writer needs to test their code roughly every hour or after every major change and if it takes several minutes to the test the software then the coder becomes reluctant to perform these tests which then allows bugs to creep in. Further, if we define every object then our arguments can sometimes explode into 500 lines which is too many lines for a rational human being to understand.

For these reasons we need to come up with some relevance rules to weed out sentences for which we have good reasons will not change the consistency-value of our arguments.

Definition 1: Standard Sentence

A sentence is standard iff it is composed of only relations, and abbreviations which refer to nouns. It may also have negation signs and abbreviations may refer to an adjectives
but only if the adjective is the object of the J relation. Abbreviations may also refer to adverbs but only if they are in the object position of the V relation. It must be stressed that the relation need not be atomic.

**Definition 2: Non-standard Sentence**

A sentence is not standard iff it has any of the following types of words:

- a) determiners,
- b) adjectives which are not objects of the J relation,
- c) coordinators such as 'and' or 'but',
- d) subordinators such as 'that',
- e) relative pronouns such as 'which' and 'who',
- f) adverbs such as 'can', 'always', 'necessarily',
- g) personal pronouns or possessive pronouns,
- h) possessive nouns such as "Marilyn's" or "the dog's"
- i) nouns which are instances and which appear in apposition to their concept,
- j) relational particles such as 'to' and 'for',
- k) tensed verbs such as 'went',
- l) infinitival to
- m) present participles such as 'sitting' or 'singing',
- n) auxiliary words such as 'does' or redundant words such as 'the'.

**Definition 3: Standard Symbol**

A symbol is standard iff it may appear in a standard sentence and a symbol is not standard if it may appear in a non-standard sentence but not in a standard sentence.

**Definition 4: More Basic Sentence**

A sentence b is considered more basic than sentence c iff b has fewer non-standard symbols than c.

**Definition 5: Eliminate a Sentence**

A sentence is eliminated iff it is non-standard and it is shown to be equivalent to a single or a set of sentences which are more basic. For example, the sentence (there EX b) is not standard since it contains existential there and it is equivalent to (b EX). (b EX) is not standard either but it is more basic than (there EX b) because (there EX b) contains two non-standard symbols whereas (b EX) contains only one. When we write down that (there EX b) ≡ (b EX) and then infer (b EX), we say that (b EX) eliminates (there EX b). This is because (there EX b) will not be used when performing instantiation.

**Definition 6: Reduce to Standard Form**
The process of reducing to standard form is that process where all sentences are eliminated until only standard sentences remain.

**Definition 7: Standard Entailment**

An entailment is standard iff the definiendum is in standard form.

**Definition 8: The Key Point**

The key point is that point in the argument where one has finished the reduction to standard form and not yet begun making standard entailments.

**Definition 9: Basic Sentence**

A sentence is basic iff it is composed entirely of abbreviations, atomic relations and if the abbreviations refer to non-relations and are the object of the I or J relation, then those non-relations must be atomic.

**Definition 10: Irrelevant Abbreviation**

An abbreviation is irrelevant iff it does not appear in the argument during the key point but appears after a standard entailment has been made. An abbreviation is relevant if it appears in the argument at the key point.

**Definition 11: Irrelevant Sentence**

A sentence is irrelevant iff each of its abbreviations are irrelevant or it has the I or J relation and its subject is irrelevant.

**Relevance Rule 1**

*Do not define irrelevant sentences.*

For example, suppose our sentences at the key point are:

90. \( (z = \text{thought}) \& (y = \text{thing}) \& (q = \text{extant}) \& (x \ I \ z) \& (w \ I \ y) \& (x \ TK \ w) \& (x \ J \ q) \)

90 is the reduction of the sentence 'there is a thought which thinks something'. Any abbreviation which appears after the key point is considered to be irrelevant. So say we define thought as follows:

\( (x \ I \ z) \equiv (u \ B \ x) \)

\( x \) is a thought iff someone believes \( x \).

Now, suppose a metaphysician raised the objection: 'how can that be right? I can think the thought "I eat a round-square" even though I do not believe it.' This is simply another
meaning of the word 'thought' which has a different entailment and which we do not need to get into right now.

The u abbreviation is considered irrelevant since it does not appear at the key point in 90. However, since the sentence (u B x) does not contain the I or J relation it must be defined, since x is relevant. Now a necessary condition for being the object of the believe relation is that it be a relationship. Once again, I can just see a metaphysician raising the objection: 'how can that be right? If I believe in God, it does not follow that God is a relationship.' Again, our fictional metaphysician is referring to a different meaning of 'believe' which has a different entailment. That relation is a pragmatic relation where what we really mean is: 'I believe that God exists'. Its necessary condition would be:

\[(b \land c) \land (d \leftrightarrow c \land e) \land (e=\text{extant})\]

Once more our imaginary metaphysician can raise an objection: 'what about I believe in democracy? I can believe that democracy exists but not believe in democracy.' Again, that meaning of 'believe' would have a different definition which we do not need to get into right now.

Taking up our train of thought before our digression, the necessary condition for being the object of the B relation is that it is a relationship which is expressed as:

\[((u B x) \rightarrow (x I n)) \land (n=\text{relationship})\]

*If u believes x then x is a relationship*

(u B x) is considered a basic sentence and hence it does not have a definition. Now, when we define (x I n) since x is a relevant abbreviation we get:

\[(g=\text{relatum}) \land (h=\text{relation})\]

\[(x I n) \equiv ((x W w_1) \land (x W v_1) \land (x W u_1) \land (w_1 I g) \land (v_1 I g) \land (u_1 I h))\]

We now have six new sentences. The sentences whose subject begins with x must be defined since they do not have the relation I or J in them and one of the abbreviations is relevant, but the sentences (w_1 I g) & (v_1 I g) & (u_1 I h) need not be defined since w_1, v_1 and u_1 are not relevant abbreviations and they appear in sentences which have the I relation.

5.5. The Justification of the First Relevance Rule

Let's now ask ourselves, how do we know that if we were to define the sentences (w_1 I g) & (v_1 I g) & (u_1 I h) we would not find a contradiction? Recall that definitions state what sentences are consistent. So if a definiendum is defined as b and its definiens contains c & d & e, then it is impossible that c & d & e contradict b. Recall also that in order to obtain a contradiction the same sentence must be affirmed and denied. So if the only sentence that contains, w_1 is (w_1 I g), for example, then it is impossible to infer (w_1 ~ I g) because the definiens never implies the negation of the definiendum.
The same applies for relevance rule 2. If the definiendum is of the form \((b \: R \: c)\) and if \(b\) and \(c\) appear nowhere else in the argument other than in \((b \: R \: c)\) then the only way to get additional sentences which begin with \(b\) is to define \((b \: R \: c)\). Further, if the only way to get a contradiction is to infer \((b \sim \: R \: c)\) then it is obvious that we are not going to infer \((b \sim \: R \: c)\) by defining \((b \: R \: c)\) since, once again, the definiendum never entails its negation.

However, if is true deductively then it should also be the case that this relevance rule if ignored would also not produce a contradiction. So far for our 190 claims we proved them both with and without this relevance rule and the consistency value did not change. While 190 is not that large a number, we would like to get that number higher in the future.

5.6. The Decision Procedure for Instantiation

There are other relevance rules but they are best discussed in the decision procedure for instantiation. Now, if there are no negated sentences and the claim is falsified through category errors or because the subject contradicts the object, then instantiation is no problem. It is when there is a negated sentence that things become tricky. So in the argument: 'it is contradictory that there is a mind which is not mental', after we reduce everything to standard form we will have the following detached and hypothetical sentences:

\[(z=\text{mind})\]
\[(y=\text{mental})\]
\[(q=\text{extant})\]
\[(o=\text{relationship})\]
\[(x \: I \: z)\]
\[\sim(x \: J \: y)\]
\[(x \: B \: v)\]
\[(x \: D \: u)\]
\[(x \: J \: q)\]
\[(v \: I \: o)\]
\[(t \: J \: y) \equiv (t \: B \: r)\]

We start with the full general variable, which here is \(t\). We then check every detached abbreviation to see if it has the properties of the full general variable. However, it is almost always the case that it is the subject or the object of the original claim which ultimately is present in the contradiction. This abbreviation is \(x\). We then check all of \(x\)'s properties to see if it either has the relational property \(Jy\), since \(y\) is a constant or if it has the property being the subject of the B relation since \(r\) is a half general variable. It indeed does have the property being the subject of the B relation and hence we can instantiate \(t\) with \(b\) and because \(r\) is a half general variable and has only one property, namely, being the object of the B relation we can thus instantiate \(r\) with \(v\).
Let's now do a more fictional example so as to make it more challenging. In the following, the relations do not have their usual meanings:

1. \((y.k_1.h.o.k J d) \& (d=\text{definite})\)
2. \((e F b)\)
3. \((c V o)\)
4a. \((e Y o)\)
4b. \((b Y c)\)
4c. \((e \sim A o)\)
5. \((g \sim D u_1)\)
6. \((c B o)\)
7. \((b T o)\)
8. \((o I k)\)
9. \((f I h) \equiv ((f T j) \& (j Z k) \& (k K m))\)
10. \((f I h) \rightarrow ((f P n) \& ((q I y) \rightarrow ((f G q) \& (q W x_1)))\)
11. \((z F b_1) \rightarrow ((g_1 Z b_1) \& (z I k_1) \& (z I y) \& (b_1 K d_1) \& (p_1 T g_1))\)
12. \((t G v) \equiv (((t B o) \& (o V v)) \lor ((v W v_1) \& (v V s_1) \& (s_1 I k) \& (v I y)))\)
13. \((c_1 P d_1) \equiv ((c_1 S x) \& (x H u))\)

As you can see we have 30 abbreviations which means we have to check 435 pairs of abbreviations. So already we have a decision procedure, we just go through all 435 pairs and see if one can be instantiated for the other. But that is too slow a process and it is quite hard to determine if an error is made. We make things easier in the following ways: one, we have divided abbreviations into general and detached abbreviations and make it a rule that only general variables can be instantiated by detached abbreviations. Two, using the axiom of difference we can filter out even more pairs since no two abbreviations which exist within the same sentence can be instantiated one for the other. Third, we use the following relevance rule:

**Relevance Rule 2**

*If a definite relation appears in a detached sentence but does not appear in a hypothetical sentence then the sentence that it appears in is irrelevant for the purposes of instantiation.*

To understand why this is true, observe the following:

91. \((b R c)\)
92. \((g Q e)\)
93. \((g \sim S f)\)
94. \((d Q e) \rightarrow (d S f)\)
The only sentences which are relevant in 91-94 are sentences which contain the relations which exist in the hypothetical sentence, that is, Q and S. Because it is only with the hypothetical sentence that we can infer new sentences and because we can only obtain a contradiction if the same sentence is affirmed and negated, it therefore follows that a definite relation must at least appear twice in a set of premises in order to be relevant. This rule eliminates the following sentences as irrelevant:

(e Y o)
(b Y c)
(e ~ A o)
(g ~ D u₁)

Although this rule only eliminates one abbreviation from consideration, g, it still makes our calculation much easier. Now, one might think at this point that it will be impossible to obtain a contradiction. After all, no sentence is negated, so how are we going to obtain a negated sentence which will be both affirmed and denied? The answer is that we might detach two sentences which belong to mutually exclusive categories, or violate one of our axioms. Our final method for making things easier is that we ignore the half general variables. For example, in the following sentence,

(f I h) ≡ ((f T j) & (j Z k) & (k K m))

since h is constant and because only f appears on both sides of ≡, it is the sole full general variable. If we cannot instantiate f with a detached abbreviation then it will not allow us to make any inferences if we succeed in instantiating a half general variable. We now have only seven full general variables:

f b₁ z c₁ d₁ v t

To check against five detached abbreviations b, e, o, c, u₁ and also three universals which almost never instantiate anyway. We will discuss our decision procedure for eliminating universals from consideration later. Our first step is to categorize all of the relational properties of each full general variable. So f has two relational properties which are:

αIh, αT

And since they appear on either side of ≡, a detached abbreviation may have either one of those relational properties, in order to begin the process of determining whether instantiation will lead to detachment. So below a comma separates relational properties into property sets or individuals. Unfortunately, in this example, the only variable which has two properties which belong to the same set is v which is why there is no comma separating αIy αW.

f - αIh, αT
z - αF
b₁ - Fa
t - αG, αBo, αV
v - Ga, aly αW, oVa
c₁ - αP, αS
d₁ - Pa

It should also be pointed out that t and v have 3 property sets because in the biconditional consequent t is disjunctive. However, one of these properties is irrelevant.

**Relevance Rule 3**

*If the definite relation in the general variable's property does not appear again anywhere else in the argument then it is irrelevant.*

To understand why, notice that if a sentence has a definite relation that no other sentence has then it cannot contradict a detached sentence, nor can it detach a sentence from a hypothetical sentence. Hence, it will never detach new sentences.

Our next step is to list the relational properties of the detached abbreviations. So if the detached sentences are:

(e F b)
(c V o)
(c B o)
(b T o)
(o I k)

then the detached properties are:

e - αFb
b - eFa, αTo
c - αVo, αBo
u₁ - eWa
o - cVa, bTa, αk, cBa

We do not check abbreviation k, since it is a universal and we have a decision procedure for determining when it is impossible that the instantiation of a general variable by a universal will not lead to contradiction. It goes something like this: if the word 'universal' itself, or 'instance' appears in our list of constants then we need to check it for instantiation, otherwise not. This is because any synonym of these two words or any relation which relates these words will eventually cause 'universal' or 'instance' to appear in our list of constants.

We now have 10 detached relational properties and 13 full general properties which is 130 pairs that need to be checked which is much better than the 435 that we started out with. From here we simply go through each general property and see if it also exists in our list of detached properties. We'll start with c instantiates t. t has property αB and c has property αBe, hence there is a match since c's property has every abbreviation and
relation that t’s property has. However, we must now see if instantiation rule 5 is not broken. We see that t has a relation to w which also has the property αV. Therefore, e must also have this property but it does not, hence no instantiation can occur.

There is also a match between b and f. b has property αTo and f has property αT. Recall that instantiation rule 5 is:

*If a full general variable b has a relation to a half general variable c then the detached abbreviation which has the same relational property as b must also have a relation to another detached abbreviation which has all the relational properties that c has.*

j is the half general variable to which f has the T relation. j also has the property αZ. b has the T relation to o. It therefore follows that o must also have the property αZ but it does not have this property, so no instantiation can occur.

We will not determine whether e can instantiate z. We first see that e has the property αFb and z has the property αF. So e can instantiate z. We also see that b₁ has the property Fα and that b has the property eFα, so b can instantiate b₁. Further, by instantiation rule 8, we need to translate d₁ into o₁ and g₁ into h₁. When we instantiate, translate, substitute and detach we may infer:

\[
\begin{align*}
13 & \quad (z \Rightarrow e) \land (b \Rightarrow b) \land (d_1 \Rightarrow o_1) \land (g_1 \Rightarrow h_1) & \text{IN} \\
14 & \quad (e \Rightarrow F \land b) \land ((g_1 \Rightarrow Z \land b) \land (e \Rightarrow I \land k_1) \land (e \Rightarrow I \land y) \land (b \Rightarrow K \land o_1) \land (p_1 \Rightarrow T \land g_1)) & \text{SUB 13,10} \\
15 & \quad (g_1 \Rightarrow Z \land b) \land (e \Rightarrow I \land k_1) \land (e \Rightarrow I \land y) \land (b \Rightarrow K \land o_1) \land (p_1 \Rightarrow T \land g_1) & \text{MP 2,14} \\
16 & \quad (g_1 \Rightarrow Z \land b) & \text{&E 15} \\
17 & \quad (e \Rightarrow I \land k_1) & \text{&E 15} \\
18 & \quad (e \Rightarrow I \land y) & \text{&E 15} \\
19 & \quad (b \Rightarrow K \land o_1) & \text{&E 15} \\
20 & \quad (p_1 \Rightarrow T \land g_1) & \text{&E 15}
\end{align*}
\]

We now update our list of properties as follows: e and b have gained two properties, and we have three new abbreviations p₁, g₁ and o₁. Hence our new list is:

\[
\begin{align*}
e & \quad \alpha Fb, \alpha I k_1, \alpha y \\
b & \quad \alpha Fb, \alpha To, \alpha K o_1, g_1 Z a \\
c & \quad \alpha V o, \alpha Bo \\
u_1 & \quad \alpha W a \\
o & \quad \alpha V a, b T a, \alpha k, c B a \\
o_1 & \quad \alpha K o_1 \\
g_1 & \quad \alpha Z b, p_1 T a \\
p_1 & \quad \alpha T g_1
\end{align*}
\]

We can now use a new relevance rule to eliminate some of the general variables from consideration:

**Relevance Rule 4**
If a full general variable is only relevant because of abbreviations which have already instantiated it, then it is no longer relevant.

So, z and b₁ are relevant because e and b have the definite relation F but since z and b₁ have already been instantiated by e and b, then they are no longer relevant. We can thus eliminate z and b₁ from our table:

f - αl₁h, αT
f - αG, αBo, αV
v - Ga, αly αW, αVa
c₁ - αP, αS
d₁ - Pa

Relevance Rule 5

If none of the full general variables in a hypothetical sentence are relevant, then the hypothetical sentence is irrelevant.

Using relevance rule 5, the remaining relevant hypothetical sentences are:

( f l h ) ≡ (( f T j ) & ( j Z k ) & ( k K m ))
( f l h ) → (( f P n ) & (( q I y ) → (( f G q ) & ( q W x₁ ))))
( t G v ) ≡ ((( t B o ) & ( o V v )) ⊻ (( v W v₁ ) & ( t V s₁ ) & ( s₁ I k ) & ( v I y ))))
( c₁ P d₁ ) ≡ (( c₁ S x ) & ( x H u ))

We can thus take a second look at our list of detached sentences and determine if any are now irrelevant using relevance rule 2:

( e F b )
( e V o )
( c B o )
( b T o )
( o I k )
( g₁ Z b )
( e I k₁ )
( e I y )
( b K o₁ )
( p₁ T g₁ )

Only ( e F b ) and ( e I k₁ ) are irrelevant, hence our revised set of relevant detached properties is:

e - αly
b - αTo, αKo₁, g₁Za
c - αVo, αBe, αVo
We now have 15 detached properties but 7 of them have already been checked against the general properties. Hence, we have 8 new detached properties to check against 11 sets of general properties which is 88 pairs.

We will first start with whether e can instantiate v since both have the property αIy. However, in the same property set which contains αIy there is also the property αW which e does not have. Hence, v cannot instantiate e.

Moving on, we see that the property of αT of f is contained with the property of p₁ αTg₁, hence, we may instantiate f with p₁ but in order to detach (f I h), we need to be able to instantiate j, k, and m. We start with j. j has the property Tα and αZ and we see that g₁ has these properties, but there is a second requirement. g₁ must have the relation Z to another abbreviation which is both subject of the K relation and object of the Z relation. We see that b has these properties, hence b can instantiate k. m has only one property and o₁ has it, so we can now use instantiation, conjunction introduction, substitution and ≡ elimination, translation and MP as follows:

\[
\begin{align*}
21 & \quad (f \Rightarrow p₁) (j \Rightarrow g₁) (k \Rightarrow b) (m \Rightarrow o₁) \quad \text{IN} \\
22 & \quad (p₁ T g₁) \& (g₁ Z b) \& (b K o₁) \quad \&I 20,16,19 \\
23 & \quad (p₁ I h) = ((p₁ T g₁) \& (g₁ Z b) \& (b K o₁)) \quad \text{SUB 21,8} \\
24 & \quad (p₁ I h) \quad \equivE 22,23 \\
25 & \quad (n \equiv q₁) (x₁ \equiv y₁) \quad \text{TR} \\
26 & \quad (p₁ I h) \rightarrow (((p₁ P q₂) \& ((q I y) \rightarrow ((p₁ G q) \& (q W y₁)))) \quad \text{SUB 25,9} \\
27 & \quad ((p₁ P q₁) \& ((q I y) \rightarrow ((p₁ G q) \& (q W y₁)))) \quad \text{MP 24,26} \\
28 & \quad ((p₁ P q₁)) \quad \&E 27 \\
29 & \quad ((q I y) \rightarrow ((p₁ G q) \& (q W y₁))) \quad \&E 27
\end{align*}
\]

We can now add the properties αIh and αPq₁ to the set of properties that p₁ has. It is very important to note that we do not add the property αGq because that is found in the consequent of a conditional sentence. We also have gained a new full general variable, namely, q. So we would add αIy, as its property. It is also important to point out that we had to employ instantiation rule 6 which is:

If a sentence is detached from a hypothetical sentence which contains an abbreviation which was once modified by α₄ then each time it is detached the indefinite abbreviation must be translated into a new abbreviation.

Because n is in the sentence (p₁ P n) which is in the consequent of

\[(f I h) \rightarrow ((f P n) \& ((q I y) \rightarrow ((f G q) \& (q W x₁)) \& (o V q)))\]
and because \( n \) does not appear in the antecedent, it is therefore not a general variable but an indefinite abbreviation. The same goes for \( x_i \). Each time a sentence is detached containing an indefinite abbreviation, the abbreviation must be translated into a new abbreviation. Hence, our updated list of detached properties is:

\[
\begin{align*}
\text{e} & - \alpha W_{u_1}, \alpha y \\
\text{b} & - \alpha T_0, \alpha K_{o_1}, g_i Z_a \\
\text{c} & - \alpha V_0, \alpha B_0 \\
\text{u}_1 & - \epsilon W_x \\
\text{o} & - c V_a, b T_a, \alpha l_k, c B_a \\
\text{o_1} & - \alpha K_{o_1} \\
\text{g}_i & - \alpha Z_b, p_{1} T_a \\
\text{p}_1 & - \alpha T_{g_1}, \alpha l_h, \alpha P_{q_1}
\end{align*}
\]

The hypothetical sentences

\[
\begin{align*}
(f I h) & \equiv ((f T j) \& (j Z k) \& (k K m)) \\
(f I h) & \rightarrow ((f P n) \& (q I y) \rightarrow ((f G q) \& (q W x_i) \& (o V q)))
\end{align*}
\]

only had one full general variable, namely, \( f \). \( f \)'s properties were \( \alpha l_h, \alpha T \) and none of our detached abbreviations have those properties, so by relevance rule 4 \( f \) is now irrelevant. And because \( f \) is the only full general variable in the aforementioned sentences, by relevance rule 5 both are now irrelevant. Because those were the only hypothetical sentences which contained the relation \( T, Z \) and \( K \), any detached sentences with those relations are now irrelevant due relevance rule 2. That means the detached sentences \((b T o), (g_i Z b), (b K o_1), (p_1 T g_i)\) are no longer relevant. Our current list of relevant detached sentences are:

\[
\begin{align*}
(c V o) \\
(e W u_1) \\
(c B o) \\
(o I k) \\
(e I y) \\
(p_1 I h)
\end{align*}
\]

And our list of relevant detached properties is:

\[
\begin{align*}
\text{e} & - \alpha W_{u_1}, \alpha y \\
\text{c} & - \alpha V_0, \alpha B_0 \\
\text{u}_1 & - \epsilon W_x \\
\text{o} & - c V_a, \alpha l_k, c B_a \\
\text{p}_1 & - \alpha l_h, \alpha P_{q_1}
\end{align*}
\]

Now, that we have ruled out some detached properties as irrelevant, let's go back to the general properties and see if we can rule out some more. The current list of these properties is:
t - aG, aB, aV
v - Ga, aly aW, oVa
c₁ - aP, aS
d₁ - Pa
q - aly

And the current set of hypothetical sentences is:

\[
((q \land y) \rightarrow ((p₁ \land G e) \land (e \land W y₁) \land (o \land V q)))
\]
\[
(t \land G v) \equiv ((t \land B w) \lor ((v \land W v₁) \land (t \land V s₁) \land (s₁ \land I k) \land (v \land I y)))
\]
\[
(c₁ \land P d₁) \equiv ((c₁ \land S x) \land (x \land H u))
\]

Although the G relation appears nowhere in the detached properties, it appears in two different hypothetical sentences, so it is still relevant. But the S relation only appears once, so the property αS is now irrelevant due to relevance rule 3. All other general properties are relevant. One can also see that the third hypothetical sentence is irrelevant because one side of its main connective there is no definite relation which is repeated elsewhere in the argument. So if we were to detach ((c₁ \land S x) \land (x \land H u)) it could not help us obtain a contradiction. This relevance rule, however, is too tricky to program for so we'll ignore it for now.

We now have 9 general properties to check against 2 new detached properties but one turned out to be irrelevant so we only need check αIh, which is not amongst our list of general properties. However, we now must check all detached properties against our new general property which is αly. We see that e does, in fact, have this property. Recall that v also has this property but it was already checked that e cannot instantiate v. It has not been checked whether e can instantiate q and it can.

Hence after using instantiation, substitution and modus ponens we get the following sentences:

31 \((e \land I y) \rightarrow ((p₁ \land G e) \land (e \land W y₁) \land (o \land V q)))\) SUB 29,30
32 \((p₁ \land G e) \land (e \land W y₁) \land (o \land V e)\) MP 18,32
143 \((p₁ \land G e)\) &E 32
32b \((e \land W y₁)\) &E 32
32c \((o \land V e)\) &E 32

\(p₁\) has gained the property αGe, e has gained the property \(p₁\land G α\) and αWy₁, and o has gained the property αVe. The hypothetical sentence \((e \land I y) \rightarrow ((p₁ \land G e) \land (e \land W y₁) \land (o \land V q)))\) is now irrelevant due relevance rule 4 and 5 since only one detached abbreviation was capable of instantiating it.

We have gained no new general variables, so our only task is to determine whether or not our new detached properties match one of the general properties. We see that there is a match between t and \(p₁\) in that t has the property αG which \(p₁\) also has. v in the antecedent of the biconditional
\((t \text{ G v}) \equiv (((t \text{ B w}) \& (w \text{ Vv}) \& ((v \text{ W v}_1) \& (t \text{ V s}_1) \& (s_1 \text{ I k}) \& (v \text{ I y}))) \\equiv (((t \text{ B w}) \& (w \text{ Vv})) \& ((v \text{ W v}_1) \& (t \text{ V s}_1) \& (s_1 \text{ I k}) \& (v \text{ I y})))\)

only has one property so whatever abbreviation \(p_1\) has the \(G\) relation to may also instantiate \(v\) which in this case is \(e\). Using instantiation, substitution and \(\equiv\) elimination we may infer:

\[
\begin{align*}
33 & \quad (t \Rightarrow p_1) (v \Rightarrow e) & \text{IN} \\
34 & \quad (p_1 \text{ G e}) = (((p_1 \text{ B o}) \& (o \text{ V v})) \& ((w \text{ W v}_1) \& (p_1 \text{ V s}_1) \& (s_1 \text{ I k}) \& (e \text{ I y}))) & \text{SUB 11,33} \\
35 & \quad (((p_1 \text{ B o}) \& (o \text{ V e})) \& ((e \text{ W v}_1) \& (p_1 \text{ V s}_1) \& (s_1 \text{ I k}) \& (e \text{ I y}))) & \equiv \text{E 143,34}
\end{align*}
\]

We now have only one full generable variable which is \(p_1\). We see that \(c\) has one of the properties in one of \(p_1\)'s property sets, namely, \(\alpha V\). But \(p_1\) has a relation to a half-general variable, \(s_1\), which has one more property in addition to \(V\alpha\), namely \(\alpha I k\). \(c\) has a relation to \(o\). In order for an instantiation to occur, \(o\) must also have the property \(\alpha I k\) and it does. The other half-general variable in the above hypothetical sentence is \(v_1\) but it only has one property and we see that \(y_1\) has this property, so we may make another instantiation. We can thus make the following inferences:

\[
\begin{align*}
36 & \quad (v_1 \Rightarrow y_1) (p_1 \Rightarrow c) (s_1 \Rightarrow o) & \text{IN} \\
37 & \quad ((c \text{ B o}) \& (o \text{ V e})) \& ((e \text{ W y}_1) \& (c \text{ V o}) \& (o \text{ I k}) \& (e \text{ I y}))) & \text{SUB 36,35} \\
38 & \quad ((e \text{ W y}_1) \& (c \text{ V o}) \& (o \text{ I k}) \& (e \text{ I y}))) & \&I 3,7,18,32b
\end{align*}
\]

From here we just use basic statement logic to obtain the contradiction. Also, we do not use De Morgan's Law, even though most logicians would, for reasons which we will discuss when we come to statement logic.

\[
\begin{align*}
39 & \quad \sim((c \text{ B o}) \& (o \text{ V e})) & \&E 38,37 \\
41 & \quad (c \text{ B o}) \& (o \text{ V e}) & \&I 32c, 5 \\
42 & \quad \sim((c \text{ B o}) \& (o \text{ V e})) \& ((c \text{ B o}) \& (o \text{ V e})) & \&I 39, 41
\end{align*}
\]

It should also be noted that we could have used 41 to detach the negation of \((e \text{ W y}_1) \& (c \text{ V o}) \& (o \text{ I k}) \& (e \text{ I y})). However, negating general variables is not all that easy and is something that we do not like doing for reasons which will be discussed later.
6. The Irregular Equivalences

6.1. The Equivalence, Conditional, Inference Rules Distinction

We need to make clear the distinction between an equivalence rule, a conditional rule and an inference rule. In many logical texts we will see lists of rules called inference rules which in fact are equivalence or conditional rules.

95. $b$ is an inference rule iff $b$ allows $c$ to believe that $d$ is true at time 2 and $c$ was agnostic about the truth-value of $d$ at time 1.

96. $b$ is an equivalence rule iff $b$ allows $c$ to believe that $d$ and $e$ are equivalent regardless of whether or not they are true or false.

97. $b$ is a conditional rule iff $b$ allows $c$ to believe that $d$ follows from $e$ but not vice-versa even though $d$ may be true or false.

For example, transitivity is sometimes referred to as an inference rule but it is a conditional rule. So 'if Gottlob is left of Gottfried and Gottfried is left of Bertrand, then Gottlob is left of Bertrand'. This rule does not allow me to infer that Bertrand really is left of Gottlob it only allows me to believe that it is true that Bertrand is left of Gottlob if I already believe that Gottlob is left of Gottfried and Gottfried is left of Bertrand. This distinction is of the utmost importance because if we are going to be competent logicians then at the very least we must understand what an inference is.

Henceforth, we will lump equivalence and conditional rules together for simplicity's sake. We divide our equivalence rules into four categories:

1. Irregular syntax rules
2. Irregular definitions
3. Instantiation rules
4. Statement logic lemmas

It turns out that the irregular definitions have to be done in a very precise order. So we need another set of rules:

5. Decision procedure for the irregular definitions
And finally, we simply cannot compute the consequences for every object. That slows down our machine enormously and leads to an explosion of symbols which is too hard for a rational human being to understand. Hence, we need rules for filtering out irrelevant objects.

6. Relevance rules

Finally, if we're dealing with natural language then we're also dealing with pragmatic sentence. So we also have a set of rules to govern these:

7. Pragmatic rules
6.2. Irregular Syntax Rules

6.2.1. Rule: Redundancy (RD)

We're actually going to start with two rules which are not irregular syntax rules. The first step of NLL is to delete words which do not have a correspondent in artificial language. The reason is usually because some other word serves to convey the meaning. So in: 'she is a woman', the word 'is' informs us that 'she' is an instance of the universal 'woman'. The 'a' is simply redundant and can be eliminated. Or in: 'a particular man walked by', 'particular' is redundant since the word 'a' informs us that an indefinite instance of man walked by. Another very important redundant word is 'same'. So in 'Brad and Angelina appeared in the same film', 'same' is redundant because the constant which will eventually refer to 'film' will be the same and Brad and Angelina will both have a relation to that constant. This sentence will eventually reduce to:

(b API c) & (d API c) & (c I f) & (c J g) & (b=brad) & (d=angelina) & (appeared in=API) & (f=film) & (g=definite)

We do not need the word 'same' because it is hardcoded into our symbols that c is the same as c. We will explain the meaning of (c J g) & (g=definite) in section 6.4.2.

6.2.2. Rule: Substitution (SUB)

The next step of NLL is to abbreviate nouns and adjectives with single letters and relations with capital letter abbreviations, sometimes longer than one letter. Hence, the following equivalence:

(julius caesar is a number) ≡ (julius caesar is number)

RD

(julius caesar is number) ≡ (b I c)

SUB

We then label what the abbreviations stand for in the first sentence after the claims as ID.

(b=julius caesar) & (is=I) & (c=number)

ID

It is important to keep in mind that sometimes these abbreviations actually are necessary for making inferences. For instance, individual persons have necessary conditions,
usually that they are men or women. When we state one of the necessary conditions for being Julius Caesar, namely, that he is a man, we do so as follows:

\[
((b=\text{julius caesar}) \rightarrow (b \ I \ c)) \ & \ (c=\text{man})
\]

If \(b\) is an abbreviation of julius caesar then \(b\) is an abbreviation of man \([a]\) and \(c\) is an abbreviation of man.

We then later use the sentence \((b=julius \ caesar)\) to detach \((b \ I \ c)\).

We now move on to the irregular syntax rules proper.

6.2.3. Rule: Concept Instance Apposition Elimination (CIA)

Observe:

98. Ther philosopher Leibniz spoke iff Leibniz spoke and Leibniz is a philosopher.

Notice that in 98 we're not eliminating words but introducing more words. All we are doing is saying that this syntactical construction where a concept noun is followed by an instance of that concept is equivalent to a more verbose construction. These are not definitions but eliminating certain sentence constructions in favor of a basic form. In our formal system this rule appears as follows:

\[
(b \ c \ R \ d) \equiv ((c \ R \ d) \ & \ (c \ I \ b))
\]

What's going on here is that the computer has already obtained the information that both \(b\) and \(c\) are nouns and further, that \(c\) is an instance of \(b\). So on the right side of \(\equiv\) the sentence \((c \ I \ b)\) is pronounced as \(c\) is an instance of \(b\). In the first sentence \((c \ R \ d)\), the \(R\) relation can be replaced with whatever specific relation occurs in the sentence to be eliminated. So 98 would be formalized as:

99. \(((b \ c \ SP) = ((c \ SP) \ & \ (c \ I \ b))) \ & \ (c=\text{leibniz}) \ & \ (b=\text{philosopher})\)

It should also be noted that when 'the' modifies a proper name it is redundant which is why it is superscripted with 'r'.

6.2.4. Rule: Predicative Complement Insertion (PCI)

100. I saw a green Martian iff I saw a Martian and he was green.

Again, notice that no word is being eliminated or defined, we're just changing the place of the adjective in the sentence. Consequently, these cannot be counted as definitions. This rule is formalized as follows:
101. \((b \mathbin{R} c \mathbin{d}) \equiv ((b \mathbin{R} d) \& (d \mathbin{J} c))\)

In 101 the computer remembers that \(c\) is an adjective and it eliminates it by simply putting the noun it modifies in the subject position of the \(J\) relation and the adjective in the object position. It should also be noted that given the observation of trope theorists we plan to rewrite rule 101 as:

102. \((b \mathbin{R} c \mathbin{d}) \equiv ((b \mathbin{R} d) \& (d \mathbin{J} e) \& (e \mathbin{I} c))\)

The reason for the rewrite is that there are instances of green. So 102 would be rewritten as:

\((i \mathbin{SEE} c \mathbin{d}) \equiv ((i \mathbin{SEE} d) \& (d \mathbin{J} e) \& (e \mathbin{I} c)) \& (c=\text{green})\)

which is pronounced as: 'I see a green d iff I see d and d is an instance of green. It is spelled out elsewhere in the argument that d is an instance of 'Martian'. In our current list of arguments this rule is called ADJ E, which stood for adjective elimination. But we decided that we were really not eliminating adjectives but just moving them around.

6.2.5. Rule: Relation Division RDA, RDB, RDC

103. Ulysses is the father of Telemachus.

In most logics which deal with relations, the word 'of' is taken to be a unit with the word preceding it. Copi is one such example. So most logicians would symbolize: 'Ulysses is the father of Telemachus' as: \((b \mathbin{FA} c) \& (b=\text{ulysses}) \& (c=\text{telemachus})\). We do not do things that way. This is because we want to be able to make the enormously trite inference: 'If Ulysses is the father of Telemachus, then Ulysses is a father.' We are able to make this inference by taking the word 'of' to be just as significant a relation as the word 'has'. Hence, 103 reduces to:

104. \((b=\text{ulysses}) \& (c=\text{telemachus}) \& (d=\text{father})\)

105. \((b \mathbin{I} e \mathbin{OF} c) \quad \text{SUB 1,2}\)

106. \((b \mathbin{I} e \mathbin{OF} c) \& (e \mathbin{I} d) \quad \text{DF the}\)

107. \((b \mathbin{I} e \mathbin{OF} c) \quad \& \text{E 4}\)

Now in order to define the \(OF\) relation we need to separate it from the I relation by taking the subject of the I relation and making it the subject of the OF relation like so:

108. \((b \mathbin{I} e) \& (b \mathbin{OF} c) \quad \text{RDA 4}\)

'Of' is almost always the converse of some 'has' relation. So in our dictionary, the OF
relation is simply defined as: \((b \text{ OF } c) \equiv (c \text{ HAF } b)\), where HAF is pronounced as 'have'. Further, the object of HAF must be a father. We call this Relation Division Type A. The other type of relation division is where the object of the first relation becomes the subject of the second relation, but the subject of the first relation is not also the subject of the second relation:

\[(b \text{ R } c \text{ Q } d) \equiv ((b \text{ R } c) \& (c \text{ Q } d))\]

We call this relation division type b. Here Q is just a variable relation. We have not used this rule yet but we plan to in the future. And finally there is relation type c which is where the subject and the object of the first relation must also be the subjects of the second relation, like so:

\[(b \text{ R } c \text{ Y } d) \equiv ((b \text{ R } c) \& (b \text{ Q } d) \& (c \text{ Q } d))\]

With the 'punch' relation we must use relation division type c. For example, 'if I punch Aristotle in the club, it follows that I and Aristotle are in the club.' It could be the case that relation division type b is very rare. I have trouble thinking of any examples. Much more common is relation division type d: "If I see Aristotle in the lake, then Aristotle is in the lake and it is contingent that I am in the lake". However, this type of relation division requires using embedded relationships so we're going to delay them until their proper place. It should also be noted that relation division can only be performed on detached sentences.

### 6.2.6. Hard Coded Rule: Order of Negation

109. It is not the case that all of the Beatles spoke quickly to the green Martian during the party while he sang a song \(\vdash\) all of the Beatles spoke to the green Martian during the party but not while he sang a song.

110. It is not the case that all of the Beatles spoke quickly to the green Martian during the party \(\vdash\) all of the Beatles spoke quickly to the green Martian but not during the party.

111. It is not the case that all of the Beatles spoke quickly to the green Martian \(\neg \vdash\) (all of the Beatles spoke to the green Martian but not quickly \(\forall\) not all of the Beatles spoke quickly to the green Martian.)

112. It is not the case that all of the Beatles spoke to the green Martian \(\neg \vdash\) not all of the Beatles spoke to the green Martian.

113. It is not the case that John Lennon spoke quickly to the green Martian \(\neg \vdash\) John Lennon spoke to the green Martian but not quickly.
114. It is not the case that John Lennon spoke to the green Martian \( \neg \) John Lennon spoke to the Martian but he wasn't green.

The biconditional antecedent in 111 is equivalent to an exclusive disjunction because to my mind it seems that it is not really clear what you are negating if an adverb and a non-definite determiner are present in the same sentence. It is very rare that we make use of these rules in this logic but it does happen, particularly when defining 'no' and 'many'. What the above entailments are meant to demonstrate is that we have some background beliefs about what is negated when we negate an entire sentence. In short, there is an order of negation, meaning if b, c and d are present then b is negated, if c and d are present, then c is negated and so on.

**Definition: Non-Definite Determiner**

Any determiners except 'the' is a non-definite determiner.

These rules can be stated in a succinct form as follows:

115. Sub-clauses are negated before prepositional phrases which are negated before either non-definite determiners or adverbs which are negated before adjectives which are negated before relations.

To put the above in a list format we write:

Sub-clauses, prepositional phrases, (adverbs or non-definite determiners), adjectives, relations.

If one wants to break these rules then one would do so with something like: 'all of the Beatles spoke to non-green the Martian during the party.

**6.2.7. Rule: Negation Transfer (NT)**

Because we sometimes divide relations we have to be careful about what the negation sign \( \neg \) is actually negating. So in 'all men do not exist on Mars in 2017', if we were to formalize that as:

\[
(\text{all } b \sim \text{EX ON } c \text{ INB } d) \land (b=\text{man}) \land (c=\text{mars}) \land (d=2017)
\]

and divide the relation as:

\[
(\text{all } b \sim \text{EX ON } c \text{ INB } d) \equiv ((\text{all } b \sim \text{EX}) \land (\text{all } b \text{ ON } c \text{ INB } b))
\]
We would get the false result that 'all men do not exist' and 'all men are on Mars in 2017'. To avoid this invalid inference, we need to transfer the negation sign to the relation that it is actually negating like so:

\[(\text{all b EX} \sim \text{ON c INB d})\]

Now, we can divide the relations without making an invalid inference.

### 6.2.8. Rule: Prepositional Relation Transfer (PRT)

**Definition 1: Main Relation**

The main relation is the relation found in the main clause which is not a prepositional relation.

**Definition 2: The Prepositional Relations**

The prepositional relations are defined simply as a list. We have to go through all of the relations in our dictionary and specifically state whether or not it is a prepositional relation or not. We do not now have an exhaustive list of the prepositional relations but the following will do:

- **INM** - (in) a group of particles are in a set of points
- **DUR** - (during)
- **T** - (at) relates a thing to a moment
- **P** - (at) relates a thing to a possible world
- **OF** - (of) loosely associated with possession
- **INB** - (in) relates a group to a member, e.g. Obama is in the democratic party.

A good rule of thumb is that aside from 'of', if the preposition is spatio-temporal then it is a prepositional relation. It should also be pointed out that quite often a typical preposition is simply part of the main relation. For example, in 'I spoke with Bertrand', 'with' is part of the relation 'spoke' and is not a prepositional relation.

**Definition 3: Other Subordinate Relations**

If a relation is not the main relation and it is not a prepositional relation then it is either a past participle or a gerund such as "I saw Anne Hathaway sitting on the bench" or "I spoke with a man born on Mars".

If the sentence does not begin with the word 'every' or 'no' and the first relation is a prepositional relation then each relation whether prepositional or not between the first relation and the main relation but not including the main relation can be placed after the
main relation or after the relations which come after the main relation. Further, if the noun immediately before the main relation is not the subject of that relation then the object of the main relation becomes the subject. For example,

116. In a hole on the bump on the log at the bottom of the sea lives a fish.

is equivalent to:

117. A fish lives in a hole on the bump on the log at the bottom of the sea.

But in the following sentence the main relation is 'has' and the noun immediately before it is the subject of that relation and hence 'a shop' does not get transferred to the subject position:

118. In a town by the beach a surfer has a shop

is equivalent to:

119. A surfer has a shop in a town by the beach.

Similarly, in the following sentence, there is a prepositional relation both before and after the main relation:

120. In a town by the beach lives a surfer in a two story building.

is equivalent to:

121. A surfer lives in a two story building in a town by the beach.

Further, if a subordinate relation should come between the prepositional relation then because the rule includes every relation between the prepositional relation and the main relation, the subordinate relation as well gets transferred:

122. In a town, populated with elves lives a surfer in a two story building.

is equivalent to:

123. A surfer lives in a two story building in a town, populated with elves.

124. Manyn men in the van by the bridge wore a black shirt.

is equivalent to:

125. Manyn men wore a black shirt in the van by the bridge.
125 is certainly awkward but recall that only the initial natural language sentence need be unawkward, which is to say, eloquent. By performing prepositional relation transfer our calculations become much easier because it is much easier to distinguish the main relation from the prepositional relation if the main relation is always first. Also note that 125 would not be legal if the 'many\textsuperscript{th}' were replaced with 'every' or 'no'.

6.2.9. Subordinate Relation Prohibition

If a subordinate clause which has a prepositional relation precedes the main clause then the prepositional relation is not transferred. This is called a prohibition rather than a rule because it does not result in an equivalence. For example, in

126. A town, replete with elves of the meaner sort, struggles with its identity.

Even though 'of' precedes the main clause it is not transferred to the end since it is embedded in the subclause beginning with 'replete'.

6.2.10. Remarks on Tense

With the exception of the philosophy of time, the metaphysician is not particularly interested in tense. We are more interested in statements which are always true, not in statements which are true at one time and not another. While it is true that we believe that if the logician brushes a problem under the rug by simply transforming a word into another without using a rule then they are bound to make mistakes, we are going to make an exception with tense. We will be routinely transforming tensed verbs into tenseless verbs without using a rule in this essay. We would one day like all of our symbolic transformations to be done according to a rule, but this is one problem that we believe we can now brush under the rug without bad consequences.
6.3. Irregular Definitions

6.3.1. The Definition of 'which'

127. I saw the house which was white iff I saw b and b is the house and b is white.

I think most logicians would say that 127 employs an axiom but I believe it is a definition of the word 'which'. In 127 we define 'the' and 'which' at the same time for clarity's sake but in our logical language they are defined separately. In any case, there is a class of words which are eliminated in highly unusual ways. It's not that we need to assume that 'which' is indefinable and it is eliminated with an axiom, rather they just have strange equivalences. With 'which' the formal definition is as follows:

\[(b \hspace{1mm} R \hspace{1mm} c \hspace{1mm} \text{which} \hspace{1mm} Q \hspace{1mm} d) \equiv ((b \hspace{1mm} R \hspace{1mm} c) \hspace{1mm} \& \hspace{1mm} (c \hspace{1mm} Q \hspace{1mm} d))\]

So 127 would be:

\[
((i \hspace{1mm} \text{SEE} \hspace{1mm} b \hspace{1mm} \text{which} \hspace{1mm} J \hspace{1mm} c) \equiv ((i \hspace{1mm} \text{SEE} \hspace{1mm} b) \hspace{1mm} \& \hspace{1mm} (b \hspace{1mm} J \hspace{1mm} c))) \hspace{1mm} \& \hspace{1mm} (c=\text{white})
\]

The fact that b is an instance of 'house' would be spelled out elsewhere in the argument.

6.3.2. The Definition of Plural Nouns

Although we might change this in the future, currently, the information that a noun is plural is put in the determiner that modifies it. For example,

128. I saw some woman.
129. I saw somep women.

The fact that 'woman' is a singular noun in 128 is contained in the fact that 'some' is used rather than 'somep'. In 129, the fact that 'women' is plural is contained in the fact that 'somep' is used rather than 'some'. For this reason, all plural nouns are simply converted to singular nouns.
6.3.3. The Definition of Existential 'There'

Defining 'there' in the sentence 'there are \( e \) dogs' is quite easy. All we do is eliminate 'there' and put the object in the subject position. For example:

\[(\text{there EX } b) \equiv (b \text{ EX})\]

6.3.4. The Definition of the Coordinator 'andc'

We will call compound nouns those nouns put together with the coordinator andc. In our current arguments we did not realize that each time we use this construction we must specify whether the members of the compound noun perform the verb separately or jointly. For example:

130. Russell andc Aristotle revolutionized logic.

If we knew nothing about history, then that could mean that Russell and Aristotle by working together revolutionized logic. If we mean this then we will use the adverb 'jointly'. But it could also mean that they each revolutionized logic by themselves at different times. If that is what we mean then we use the adverb 'separately'.

131. Russell andc Aristotle separately revolutionized logic.

132. (b=russell) \& (c=aristotle) \& (d=logic)

133. (b andc c separately REVd) \hspace{1cm} \text{SUB 131, 132}

134. (b REV d T e) \& (c REV d T f) \hspace{1cm} \text{DF andc 133}

If the compound noun occurs in the object position then its elimination works like this:

135. I saw Marilyn andc Audrey separately.

136. (c=marilyn) \& (d=audrey)

137. (i SAW c andc d) \hspace{1cm} \text{SUB 135, 136}

138. (i SAW c Te) \& (b SAW d Tf) \hspace{1cm} \text{DF andc 137}

We have not used the joint andc in any of our arguments so we will pass over this for now.
6.3.5. The Definition of the Subordinator 'that'

Our software is not yet programmed to handle embedded sentences but when we get around to programming for it this is what it will look like. Suppose we wanted to formalize:

139. I believe that you area smart.

This would be reduced as follows:

140. (c=you) & (d=smart) & (e⇿c J d)  
141. (i B that c J d)  

Now we have already abbreviated the sentence (c J d) as e in 2. So all we do is simply eliminate 'that' and place e in the object position of the B relation. On the left of the ⇿ sign is always the name of the sentence and on the right is always the sentence that it refers to, for this reason we need not write: (e⇿(c J d)) since ⇿ is always the main connective.

142.  (b B e)  

DF that 141

6.3.6. The Definition of Propositional 'it' and 'that'

'it' and 'that' are defined at the same time. So far we recognize two meanings of the word 'it'. The first is just a simple pronoun as in 'I saw it', 'I felt it', 'I did it'. And 'it' can stand for a sentence or a non-sentence. The 'it' that most interests us at the moment is what we call propositional 'it'. Observe:

143. Itp isa necessaryo that you study metaphysics.  
144.  Itp isa desirable that you ignore set theory.  
145.  Itp isa intelligent to avoid continental philosophy.  

Here, 'it' refers to the sentence subordinated by 'that' or changed into a phrase with the infinitival 'to'. So in 143 'it' refers to 'you study metaphysics'. We formalize this as follows:

146. (p⇿you study metaphysics) & (b=necessary)  
147. (itp J b that p)  
148.  (p J b)  

DF itp 147

So we see in line 8 that itp refers to p so we simply replace 'itp' with p and eliminate 'that'. We should also point out that the type of necessity being used in 4 is the optative necessity. This is found in constructions like 'I must have her', 'The business must not collapse', 'The jobs must not be shipped overseas', which means that we have a very intense desire for p. Let's do another example:
6.3.7. The Definition of Infinitival 'to'

Suppose we wanted to formalize 'I want to go home'. This reduces to the more awkward: 'I want that I go home'. In other words, the subject of the main clause becomes the subject of the subordinate clause. Hence,

152. I try to learn logic

reduces to:

153. (i TRY to LRN c) & (c=logic)

whose equivalence is:

154. (i TRY to LRN c) ≡ (i TRY d) & (d ⇔ i LRN c))

6.3.8. The Definition of the Relational Particles

On some occasions we need to use relational particles. These are words like 'to' and 'for' in the following sentences: 'it is true to me that Coke tastes good', 'it is good for Plato that he taught Aristotle'. The way we handle these is that they appear in the definiendum of certain words and do not appear in the definiens. In this way they are eliminated. So the definition for 'b is desirable to c' is equivalent to 'c desires b'. Hence, the particle 'to' appears in the definiendum but not in the definiens. Relational particles, therefore, do not have a separate entry in our dictionary.

6.3.9. Brief Remarks on Modal Logic

In this system modal logic is not considered special or unusual. 'Necessary', 'contingent', 'possible' and 'impossible', as well as some others, are words just like any other which can be reduced to the atoms. We do not take the symbols □ and ◇ as basic but are considered definable and for this reason we see no need for them. Because the modal terms are definable, NLL does not take the following as axiomatic but can actually argue for them:

(□p → ◇p)
(□p → p)
Even though the modal terms are not considered to be irregular it is nevertheless necessary (pun intended) that we state their definitions now because they will be used in the following sections. First, we are presently only concerned with logical possibility. So when someone says 'you cannot walk and chew gum at the same time', it is probably physical necessity that is referred to. In 'you cannot vote if you are under 18', this is legal necessity and in 'you cannot make the first move on a guy if you are a girl', this is conventional necessity that is meant. There could be as many as 10 different types of necessity but we are not concerned about that here. Logical necessity and possibility have in common that they are related to the word consistency. Recall that consistency is the word we use to establish facts about what we will never believe at the same time so that our interlocutors can make inferences about what other beliefs we have. Because the word 'consistent' and the relation P and U are all atomic, we need to take their relationships as axiomatic. Hence:

\[(\text{b} \text{ P c}) \rightarrow (\text{b} \text{ J d}) \& (\text{d}=\text{consistent})\]

*If b exists at\[a\] possible world, then b is\[a\] consistent.*

It is very important to point out that the converse is not true:

\[(\text{b} \text{ J d}) \not\models (\text{b} \text{ P c}) \& (\text{d}=\text{consistent})\]

This is because in order to understand probability we had to prohibit the possibility of the same possible world existing at different times. Second, when we say that 'b is consistent' what we really mean is that 'b is a complex sentence which is equivalent to a set of atomic sentences, say, c & d & e & \sim f which can all be believed at the same time since it is not the case that the same sentence is affirmed and denied.

So if 'b is consistent' then it follows that it is also consistent some of those relationships are true at one time, whereas others are true at other times and if they are true at different times then they are true at different possible worlds. So suppose I were to say 'it is consistent that b is a caterpillar at time 1 and b is a butterfly at time 2'. If I were to name that conjunction c, since in our logic abbreviations can refer to anything including sets, it follows that c does not exist in a possible world but is a conjunction where some of the parts exist in one possible world and others in another possible world. For this reason, we say that c exists in a possible universe since possible universes are composed of a set of possible worlds. For this reason, we take the following as axioms:

\[(\text{b} \text{ U c}) \rightarrow (\text{b} \text{ J d}) \& (\text{d}=\text{consistent})\]

*If b [exists] in\[a\] possible universe then b is\[a\] consistent.*

\[(\text{b} \text{ J c}) \rightarrow ((\text{b} \text{ U d}) \not\equiv (\text{b} \text{ P d})) \& (\text{c}=\text{consistent})\]

*If b is\[a\] consistent then b [exists] in\[a\] possible universe
It is very important to point out that the object of the U relation must be the same as the object of the P relation. This is because it is consistent for the same relationship to exist in both a possible world and a possible universe but the same object cannot both be a possible world and a possible universe, just as an object cannot be both a point and a set of points. Hence, the following is axiomatic:

$$((b \text{ P } c) \to (b \text{ U } d))$$

But the converse is not. When it is not true that a relationship does not exist in a possible world, then we may infer that that relationship is equivalent to a set of relationships some of which are true in some possible worlds and false in others. For example, 'I ate pizza while watching TV'. This is equivalent to 'I ate pizza and watched TV at time 1, I ate pizza and watched TV at time 2, etc'. The first sentence is true at one possible world, and the second sentence is true at a different possible world. This axiom is expressed as follows:

$$((b \text{ W } c) \& (b \text{ U } d)) \to ((b \text{ W } e) \equiv ((e \text{ U } d) \& (e \text{ P } f)))$$

**If [group] b has\textsuperscript{w} [member] c and b [is true] in\textsuperscript{w} universe d then b has\textsuperscript{e} e iff e [is true] in\textsuperscript{w} universe d and e [is true] at\textsuperscript{e} [a] possible world.**

In simpler terms, if b is a set and b is true in a possible universe d, then each of its members is true at a possible world which exists in possible universe d. That should clarify consistency's relationship to possibility. We can now state the definitions of the modal terms:

- (c=consistent)
  
- (b=logically necessary) & $$((p \text{ J } b) \equiv ((p \text{ J } c) \& (\neg p \text{ J } c)))$$
  
- (b=logically possible) & $$((p \text{ J } b) \equiv (p \text{ J } c))$$
  
- (b=logically contingent) & $$((p \text{ J } b) \equiv ((p \text{ J } c) \& (\neg p \text{ J } c)))$$
  
- (b=logically impossible) & $$((p \text{ J } b) \equiv ((\neg p \text{ P } c) \& (p \text{ J } c)))$$
  
- (b=absurd) & $$((p \text{ J } b) \equiv ((p \text{ J } c) \& (\neg p \text{ J } c)))$$

The absurd property applies to sentences such as: 'the present King of France is bald', 'colorless green ideas sleep furiously', 'this sentence is false', 'the sentence is not provable', 'I ate a round-square', 'there is a barber in Seville who shaves all and only those who do not shave themselves', 'the set of all sets which are not members of themselves is a member of itself,' which is to say both their affirmations and their negations are not consistent.

### 6.3.10. Brief Remarks on the Word 'True'

To understand the meaning of 'indefinite' I first need to make some remarks on the unusual properties that the word 'true' has in our system. Observe:
155. It is true in a possible world that is not actual that Hillary won the 2016 election.
156. It is true that Hamlet is Danish.
157. In some versions of chess no later than the year 1850 it was true that black could
move first.
158. It was true in the imagination of some radio listeners during Well's War of the
Worlds broadcast that aliens had invaded Earth.

From the above sentences it should become apparent that 'true' has nothing to do with
reality. 155-158 all describe sentences which are not true in reality but are true in some
other domain. Hillary winning the 2016 election, Hamlet being Danish, black moving
first in chess, aliens invading Earth, none of these are true in reality. In our system, 'true'
and 'false' have no other meaning than:

\[(c = \text{true}) \& ((b J c Q d) \equiv ((b J e Q d) \& (b I f))) \& (f = \text{relationship}) \& (e = \text{extant})\]
\[b \text{ is}^{a} \text{ true in}^{a} d \text{ iff } b \text{ is}^{a} \text{ extant in}^{a} d \text{ and } b \text{ is}^{a}[a] \text{ relationship}\]

\[(c = \text{false}) \& ((b J c Q d) \equiv ((b J e \sim Q d) \& (b I f))) \& (f = \text{relationship}) \& (e = \text{extant})\]
\[b \text{ is}^{a} \text{ false in}^{a} d \text{ iff } b \text{ is}^{a} \text{ not extant in}^{a} d \text{ and } b \text{ is}^{a}[a] \text{ relationship}\]

Recall that the Q relation is a variable relation. Whenever we state that a relationship is
true we need to state in what domain it is true. And all we are saying is that it exists in
that domain. And recall that the most important axiom regarding existence is that 'if b
does not exist in c, then b has no properties in c'. That axiom has a lot of apparent
counterexamples, which we have not yet worked out and will not be discussed here.

It must also be stated that in the popular idiom 'this is true', is almost always a
paraphrase for 'this is true in reality'.

6.3.11. The Definition of Indefinite and Definite

In this system, each object must be either definite, indefinite or general. We will discuss
'general' when we discuss instantiation. Definite and indefinite are considered to be types
of the property 'particular'. So as to keep symbols to a minimum if an object appears in a
detached sentence or in the consequent of a conditional sentence but not in the antecedent
then it is considered to be indefinite unless explicitly stated otherwise. If an object is an
abbreviation for a word then it is definite or a constant.

The definition of definite is simply that all definite objects are not identical to each
other unless explicitly stated otherwise. So the best we can do for a necessary condition
of definite in our formal notation is:

\[(((b J c) \& (d J c)) \rightarrow (b = d)) \& (c = \text{definite})\]

If b is\(^a\) definite and c is\(^a\) definite then b is not identical to d.
Now, suppose we believe Mark Twain and Samuel Clemens are identical and definite, what do we do? We have to place this information in our dictionary as a synonymous definition:

(mark twain = samuel clemens)

We have now explicitly stated that Mark Twain is different from every other object in the universe except for Samuel Clemens. We then simply put this exception into our definition of 'definite':

\[((b \land c) \land (e \neq d) \land (e \land c)) \rightarrow (e \equiv b)) \land (c=\text{definite}) \land (b=\text{mark twain}) \land (d=\text{samuel clemens})\]

Basically, the meaning of 'definite' is hard-coded into how we manipulate our symbols. So if abbreviations b and c are definite then \((b \land c) \land (b \rightarrow c) \land (c \rightarrow b)\), which is to say, they do not refer to the same thing and one cannot be instantiated for the other. In other words, there are two types of objects in the universe: those that we believe are different and those where we're not sure that they are different. If we're certain that objects b c and d different then we put those objects into a group, we assign each of them a different abbreviation and we never assert that they are identical or that they instantiate one another. These are the definite objects.

The indefinite objects, on the other hand, might be identical to one of the definite objects or they might be identical to each other or they might even instantiate one another. It should also be stressed that 'definite' and 'indefinite' are relative to the believer, so while 'John Milton' is definite for me, he is indefinite for someone unfamiliar with English Literature.

For example, consider the following objects:

b - plasmid
c - small DNA molecule within a cell that is physically separated from a chromosomal DNA and can replicate independently (we will abbreviate this as b)
d - replicon
e - a particular protein shell existing on a particular virus
f - capsid
g - living thing
h - Anne Hathaway
i - Keira Knightley

For most people unfamiliar with cell biology objects b - f are indefinite but it is at least known that e is not instantiable since it is particular. We know that g is instantiable. h and i are definite since we know that they are not identical and they cannot be instantiated. Because h and i are definite we can infer \((h \equiv i)\). We do not know what objects b - f are, so it could be the case that \((b = c)\) or \((b = d)\) or a particular b instantiates b or a particular d instantiates b. For the record, \((b = c)\) and a particular b instantiates d
and a particular e instantiates f. No particular object which instantiates objects b - f instantiates g. And it is already known that h and i instantiate g.

In short, here is how we formalize 'indefinite':

159. If I saw a woman then it is possible I saw Anne Hathaway.

The rule regarding indefinite abbreviations is as follows:

160. If b is indefinite and if b and c do not share contradictory properties, then it is contingent that b and c are intrinsically identical.

Let's first state what we mean by intrinsically identical. Observe:

161. Lois Lane loves Superman.
162. Lois Lane does not love Clark Kent.
163. Clark Kent and Superman are identical.
164. c and d are identical is equivalent to e is a property of d iff e is a property of c.

164 expressed in natural language is slightly ambiguous so let's remove the ambiguity:

(b=identical) & ((c d J b) ≡ ((d J e) ≡ (c J e)))

161-164 are contradictory because Clark Kent has a property that Superman does not, namely, that Lois Lane loves the one and not the other. If for whatever reason that reasoning is false, then the example can be amended to 'Clark Kent has the property "was thought by Lois Lane to have done something that Superman did not"'. Clearly, not everyone will believe that Superman's and Clark Kent's properties diverge, but I certainly do and I must build a system which reflects this belief. Because of the contradiction within 127-164 we need to draw a distinction between properties which identical objects must share and properties which they share contingently. It will take a very long time before we get around to drawing that distinction so for now we won't worry about it, but will simply warn the reader that we are here advancing a temporary theory which we hope to improve later. Let's now state what we mean by contradictory properties:

7d (b=contradictory) & ((c d J e) ≡ ((c.d I f) & ((g J c.d) → (h I j)))) & (f=property) & (j=contradictory) & (h ⇿ (g J c.d))
c and d are contradictory iff c [and] d are properties and if g is a [both] c [and] d then [the relationship] h is contradictory where h stands for g is a both c and d.

In simpler language, c and d are contradictory is equivalent to if the same object were to have c and d, then that would result in a contradiction. Hence, our definition for indefinite is as follows:

(b=indefinite) & ((((c J b) & (d J e)) → (d J f)) & (d ⇿ ((g J h) & (h J j)) ≡ (c J h))) &
c is a indefinite and d is a consistent where d is an abbreviation for g is a h and h is a intrinsic iff c is a h entails that d is a contingent.

In simpler language: If c and g sharing all intrinsic properties does not result in a contradiction and c is indefinite, then it follows that it is contingent that c and g share all intrinsic properties.

6.3.12. The Definition of the Pronouns

In our dictionary, the definition of 'he' for example is written as:

\(((\text{he} \mathcal{R} \text{c}) \equiv ((\text{b} \mathcal{R} \text{c}) \& (\text{b} \mathcal{J} \text{d}))) \& (\text{d} = \text{male})\)

We should also add that 'him' has the same consequences as 'he' it's just that 'him' violates certain grammatical rules that 'he' does not. 'He' can also appear in a large number of places in the sentence. So, for instance, if the sentence were:

(he SEE b INM d)

Then it's equivalent would be:

(c SEE b INM d) \& (c \mathcal{J} e) \& (e = \text{male})

If the sentence were:

(b SEE he INM f)

Then the equivalent would be:

(b SEE c INM f) \& (c \mathcal{J} e) \& (e = \text{male})

It should also be noted that the first person pronoun is a constant. So in any argument which "I" appears we make at least once the following inference:

\(((\text{i} \mathcal{R} \text{b}) \rightarrow (\text{i} \mathcal{I} \text{d})) \& (\text{d} = \text{person})\)

'We' is somewhat more tricky since it is a group. We can have a debate about whether or not the following is a higher order sentence since technically our abbreviations are referring to groups rather than individuals in the following sentences. So 'we went to Maine in March' would be formalized as:
((we GO b INB c) & (b=maine) & (c=march))

(((we GO b INB c) ≡ ((d W e) ≡ ((e GO b INB c) & (e I f))) & (f=person)) & (b \in b) & (c \in b))

We go to b in b c is equivalent to [group] d has [member] e iff e goes to b in b c and e is a [a] person.

We should also add that it is usually the case that the pronouns are definite but not always. For instance, in 'I went on a date with a woman last night. You did? What was her name?', 'her' is indefinite to the listener but not to the speaker. For indefinite pronouns we simply superscript them with an i, like so:

(((he^i GO c) ≡ ((b GO c) & (b J d) & (b J e))) & (e=indefinite) & (d=male))

### 6.3.13. The Definition of the Possessive Pronouns

There are a lot of different ways that one object can possess another object and we do not have all of these worked out yet mostly because they are not very interesting to metaphysicians. So one object can own another object in a legal sense, just as a woman owns a car, one object can possess a body part, a group can possess a member, a universal can possess an instance, a mind can possess a belief, a desire or a sensation, a government can possess a citizen, a dead object, such as a mountain, can possess another dead object, such as a rock, etc, etc. For each of these types of possession we need to use a different superscript on the possessive pronoun. For now, we'll just do one example and we will not define the actual possessive relation.

(b SEE his d) ≡ ((b SEE e) & (b OWN e) & (e I d))

b sees his d iff b sees e and b owns e and e is a [a] d

In other words, we instantiate the universal d with an arbitrary object, here e, and then in the sentence (e I d) we state that e is an instance of d and then we also state that b owns e. As with the pronouns we need to use a different possessive pronoun if the object is indefinite than if it is definite.
6.3.14. The Decision Procedure for Reducing a Sentence to Standard Form

The decision procedure is the most important part of this logical system. Without a decision procedure there is always some uncertainty that we are forgetting a rule and consequently our arguments are not valid. Further, the decision procedure prevents a logical system from having the appearance of a shell game. If the logician manipulates their symbols by hand, there is always some suspicion on the part of the reader that they are not being honest and they are merely using those rules that help them argue for their claims. With a decision procedure and a computer that processes them, the reader does not suspect that they are being conned and can determine for themselves whether or not the arguments of the system are intuitive.

Step 1: Eliminate redundant words.

Step 2: Shorten relations and abbreviate nouns, adjectives and adverbs.

Step 3: Replace synonyms.

Step 4: Define irregular terms and eliminate irregular syntax.

Step 5: Define regular terms.

Step 6: Employ instantiation and add relevant axioms or lemmas to the premises.

Step 7: Employ statement logic.

Steps 1 - 3 I think are self-explanatory. Step 4 requires extensive explanation. The irregular terms need to be defined in a certain order but it is not entirely rigid. Still, since we are using a computer we have to choose some kind of order, though the reader needs to be aware of which elements of the decision procedure are arbitrary and which are necessary. We place irregular terms and syntax into the following categories and order:

1. Definite and indefinite articles, pronouns and possessive pronouns
2. Common name possessives
3. Proper name possessives
4. The 'and' coordinator
5. Predicative complement insertion
6. Concept instance apposition elimination
7. Eliminate parenthetical phrases
8. Relative pronouns
9. Relation division
10. Existential 'there'
11. Conditional quantifiers

**Definition: Class One Words**

The class one words are the definite and indefinite articles, the possessive pronouns, the common name possessives and the proper name possessives.

**Definition: Class Two Items**

The class two items are the relative pronouns and the rule predicative complement insertion.

**Decision Procedure Rule 1**

*If the sentence contains a conditional quantifier then use the decision procedure explained in the section on conditional quantifiers.*

**Decision Procedure Rule 2**

*Class one words must be defined before class two items.*

**Decision Procedure Rule 3**

*Relation Division must be performed after defining the class two items.*

Let's now try to understand the motivation behind rule 2. Suppose you have 'the pathetic logician ran home' which is pragmatic for 'the pathetic logician ran to their home' which would be formalized as:

165. (b=logician) & (c=pathetic) & (d=home)
166. (the c b RN their d)

If we were to perform predicative complement insertion on 166 we would get something like:

167. (the b RN their d) & (b J c)

which is false because now we're saying that the universal 'logician' is pathetic. Let's now see what happens when we try to define a relative pronoun before an indefinite article. Suppose we have: 'a man who drank some beer cursed,' after performing steps 1 - 3 we would get:

168. (b=man) & (c=beer)
169. (a b who DRK some c CUR)
We could in theory define 'who' first with something like:

170. (a b DRK some c) & (a b CUR)

But then we would have to code the computer to instantiate b with the same abbreviation in each sentence which is more difficult and might confuse the reader. It is much easier to define the indefinite article first. Here's what happens when we try to perform predicative complement insertion on a common name possessive:

171. The dog's green ball got chewed.
172. (b=dog) & (c=green) & (d=ball) & (e=chewed)
173. (the b's c d GT e)

The same problem will occur when we try to define 'the' before placing the adjective in a new position. We cannot know what is green until we have chosen an abbreviation which is to be the instance of ball. Let's now see what happens when we try to divide relations before we placed the adjective in the predicative complement position. Suppose we have: "the man sang in the grassy field."

174. (b=man) & (c=grassy) & (d=field)
175. (e SG IN c d) & (e I b)

We can now infer that (e IN d), but if the abbreviation c is in between IN and d, then this just causes more difficulty in trying to program things and it just makes our rule more complex when we try to articulate it. It is much easier to simply do the following:

176. (e SG IN d) → (e IN d)
6.4. A Special Class of Irregular Definitions: the Determiners

6.4.1. Introduction to the Determiners

The determiners are words like: 'a', 'the', 'both', 'any', 'all', 'every', 'each', 'some', 'many', 'few'. We also include numbers in this list. These words have very strange properties, are highly ambiguous and are quite difficult to formalize. On a side note, linguists usually call these words 'determiners' but for some reason in the monumental 1800 page Cambridge Encyclopedia of English grammar, the authors switched it to determinatives for no apparent reason. It seems that the linguistic community has ignored this innovation so we are sticking with determiners.

It should also be pointed out that although these words are exceedingly complex they do not have the sex-appeal that other words have and consequently readers grow exceedingly bored and frustrated when logicians/semanticists spend too much time on them. Quite frankly, this is the part of the job I hate. I realize I come across as a pedant when I analyze these words but in order to build a consistent metaphysical system we have to have a theory about these words.

As far as what all determiners have in common if we have a phrase of the form (determiner noun) and if the noun is abbreviated as b, then in the definiens of the determiner we will find a sentence of the form (c I b) where c is an arbitrarily chosen new abbreviation. We also take determiners to be synonymous with quantifiers.

6.4.2. The Definition of the Definite and Indefinite Article

The first meaning of the definite article is that it is simply redundant. Hence, in 'the' Eiffel Tower is large', once we abbreviate 'Eiffel Tower', we can simply delete 'the'. The next meaning of 'the' is the most common use of it:

\[((\text{the } b \text{ c}) = ((d \text{ I b} ) \& (d \text{ R c} ) \& (d \text{ J e} ))) \& (e=\text{definite})\]

We choose an arbitrary abbreviation, here d, and state that it is an instance of the universal modified by 'the', here b. We then state that d has the relation R to c and finally we state that d is definite. For example, a natural language reduction of 'the' is roughly:

The dog ran home iff e is a' dog and e ran home and e is a definite.
(b=dog) & (c=home) & (d=definite)
(the b RN c) ≡ ((e I b) & (e RN c) & (e J d))

If it should happen that 'the' modifies a noun in the object position, then the definition would be:

((b R the c) ≡ ((d I c) & (b R d) & (d J e))) & (e=definite)

For example:

I saw the dog iff d is\(^g\) a\(^i\) dog and I saw d and d is definite.
(b=dog) & (c=definite)
(i SEE the b) ≡ ((i SEE d) & (d I b) & (d J c))

The definition of the indefinite article is the same as the definite article except as we said earlier we do not write down that the object it modifies is indefinite because all abbreviations in detached sentences are assumed to be indefinite unless they are an abbreviation of a word or it is explicitly stated that they are definite.

We also need to mention the use of a\(^i\). In the sentence: 'this is\(^g\) a\(^i\) dog', a\(^i\) is entirely redundant. All the information we need is contained in the word 'is\(^g\)' which simply relates an instance to a universal.

In some cases 'the' is also indefinite:

Each man had the\(^i\) bit between his\(^i\) teeth.

Here, 'the\(^h\)' refers to a group of bits and it is probably the case that these bits are indefinite.

6.4.3. Definite for the Speaker but Indefinite for the Listener

Quite often it is the case that the object modified by 'the' or 'a' is definite for the speaker but indefinite for the listener. So suppose I said to my interlocutor, 'the person I was speaking to at the party went home early'. I know exactly who I was talking to but my interlocutor knows only that the object is a person and they went home early. Similarly, if I hold up a letter to a friend that I have already read but they have not and say: 'I have a letter from France,' then the letter is definite for me but indefinite for my friend. We call this indefinite\(^l\), where \(l\) stands for listener. We have not coded for this yet but when we do it would be something like this:

(j=indefinite\(^l\)) & ((b SPK c d) → ((g I e) & (g R f) & (g J h b) & (g J j d))) & (c ⇔ the\(^a\) e R f) & (h=definite)

If \(b\) speaks \(c\) to \(d\) and \(c\) is the\(^a\) e R \(f\) then \(g\) is\(^g\) [an instance of] e and g R \(f\) and g is\(^a\) definite to \(b\) and g is\(^g\) indefinite to \(d\).
### 6.4.4. On the Distinction between the Singular 'a' and the Plural 'a' 

177. Every tech guru will be awarded an\(^h\) new iPhone.
178. Every girl who sees a\(^f\) handsome prince, falls in love with hima.
179. A man walked home.
180. Have you broken your promise and flirted recently with a\(^h\) woman who was not your wife?
181. I've lived in an\(^a\) foreign country before. (This includes the background belief: 'I have lived in many\(^a\) foreign countries.)

The main difference between 179 versus 178 and 177 is that in 179 'a' refers to a singular object. The following chart will help to distinguish all three where the \(f\) stands for 'full' and the \(h\) stands for 'half' since 'a\(^h\)' modifies nouns which will become half variables and 'a\(^f\)' modifies nouns which will become full variables:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>a(^f)</th>
<th>a(^h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>refers to one thing</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>can be replaced with any(^c)</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>can be transformed into 'every'</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>modifies a noun which will exist in a hypothetical sentence after it has been reduced to standard form</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>modifies a noun which will exist in both the antecedent and the consequent after it has been reduced to standard form</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

In 178 'a\(^f\)' can be replaced with 'any', this is not the case with 'a' and 'a\(^h\)'. Further, 'a\(^f\)' can be transformed into 'every' if we make some of the verbs passive though at times this can be very awkward:

182. Every handsome prince who is seen by a girl is fallen in love with by the aforesaid girl.

Although 178 and 177 do not appear to be hypothetical sentences after 'every' and 'many\(^a\)' are defined, they will become hypothetical sentences. Once this reduction has been performed a\(^h\) will not exist in both the antecedent and the consequent whereas a\(^f\) will.

Now we haven't decided yet whether we need to superscript the indefinite article in 182 and 181 with a different letter than the one we use in 177. Recall that we only disambiguate words when in some situations word b in sentence e entails c and word d in sentence e does not entail c. For example, a\(^f\) entails 'being synonymous with "any"'.
whereas a \(^{\text{h}}\) does not. Both a\(^{\text{p}}\) and a\(^{\text{h}}\) refer to a set of objects but a\(^{\text{h}}\) appears to be slightly different. But that difference could just be because it is the fact that it exists within a hypothetical sentence.

### 6.4.5. Donkey Anaphora

The observation that the indefinite article in 178 has unusual properties was made as far back as 1328 by Walter Burley. This knowledge like much of medieval logic was forgotten and rediscovered in the 20th century and in this case it was Peter Geach who made the discovery. The sentence Geach pointed to is:

183. Every man who owns a donkey beats it.

In Burley's example, 'beat' is replaced with 'see'. The problem predicate logic has with 183 is that the abbreviation modified by the indefinite article will appear on both sides of the conditional sign and it makes more sense to quantify it with \(\forall\). But because predicate logic only has rough rules of thumb as to how to translate natural language into notation, there is no clear, orthodox explanation as to what is going on. Generally speaking, nouns modified by the indefinite article and 'some' are supposed to be quantified with \(\exists\) but why there is an exception in this case is lost on the predicate logician. In this logic donkey anaphora presents no problem because the real difference is between abbreviations which exist on both sides of a hypothetical connective and those that do not.

We now need the exact definition of a donkey indefinite article so as to distinguish it from the plural indefinite article:

b is a donkey indefinite article and b modifies g iff a noun c is modified by a universal quantifier and c exists in clause d and noun c is modified by sub-clause e and there is an indefinite noun in e which is group f and g is an unidentified member of f and g is referred to again in clause d.

From this definition it can be argued that:

If b is an abbreviation which was modified in a less basic sentence by a donkey indefinite article and b exists in a standard sentence, then b exists in both the antecedent and the consequent of a hypothetical sentence.

Using this definition we can now generate an infinite number of donkey sentences:

184. Every guy who dates a gold-digger eventually comes to despise her.
185. Every serious logician who hears an argument by Hegel, will reject it.
186. Every leprechaun who sees a rainbow, tries to reach the end of it.
187. Every dog who has a master, is obedient to them.
In 184-187 each 'a' can be replaced with 'any'. Similarly, we can transform each sentence so that 'a' becomes 'every':

Every gold-digger who dates a guy eventually is despised by him.
Every argument of Hegel's heard by a serious logician is rejected by the aforementioned logician.
Every rainbow seen by a leprechaun will cause the lephrechaun to search for its end.
Every master who has a dog is obeyed by the aforementioned dog.

6.4.6. Distinguishing Between 'a' and 'ah'

188. All of the boys at the party kissed a girl.

In 188 if the speaker of the sentence has a background belief that all of boys kissed just one girl, then most likely they would replace 'a' with 'the same' and hence 'the' would be the listener indefinite 'the'. Very rarely is it the case that the indefinite article refers to the same object when it exists in the same clause as a conditional quantifier. For example, after a short survey of 14 cases, the only plausible candidate I found was:

189. Every dance was graded on a scale of 1 to 10.

A good rule of thumb therefore is to always assume that if the indefinite article exists in the same clause as 'every' and if 'every' modifies the object and the indefinite article 'a' modifies the subject then 'a' is meant rather than 'a\text{^h}'. So far I have not yet found an instance where 'every' modifies the subject and 'a' modifies the object rather than 'a\text{^h}', though I can think of the artificial example:

77i Every judge graded that dance on a scale of 1 to 10.

To get beyond mere rules of thumb we simply ask ourselves whether each member of the group modified by 'every' has a relation to the same object or to different objects.

190. A dog went into every home on 4th street.

Although it is ambiguous whether it is a different dog going into each home or not, I think the default assumption is that it is the same dog which is going into each home.

In the following it's quite possible that the person is in the same school:

191. I am in a school teaching every day of the year.

However, when we observe more context it appears that this person is actually in different schools since elementary schools are usually quite small and it does not seem
possible that you can teach the same set of about 100 kids most every day of the year about nutrition:

192. Most every day during the school year, I am in an elementary school teaching children about their food and inspiring them to improve their diet.

In 193, on the other hand, it's quite obvious that each respondent will be buying a different truck:

193. Given the current engineering frontier, meeting these needs would raise the price of the truck, thus not every respondent in the unmet-need segment would purchase a new concept truck.

### 6.4.7. The Natural and Logical Negation of the Determiners

This rule has not yet been coded for but in natural English there is a default assumption as to what the negation of a determiner is equivalent to. These default assumptions which are not exhaustive are:

- I do not\(^n\) have a dog iff I have no dogs.
- I do not\(^d\) have any\(^n\) dogs iff I have no dogs.
- I do not\(^d\) have many\(^n\) dogs iff I have few dogs.
- I do not\(^d\) have every dog iff I have many\(^n\) dogs.
- I do not\(^d\) have some\(^p\) dogs iff I have no dogs.

When we write down the rule for these negations we write:

- DF not\(^n\) a
- DF not\(^n\) any\(^n\)

If someone falsely asserts that a certain determiner modifies a noun and their interlocutor wants to correct them but does not want them to make the usual inferences that come with negating the determiners then different syntax has to be used. The following is not exhaustive:

- I do not have just a dog, I have a lot of dogs.
- It is not the case that I have never seen a Beatle, I have in fact seen all of the Beatles.
- It is not the case that many\(^d\) senators favor the bill, in fact none of the senators favor the bill.
- It is not the case that many\(^d\) senators favor the bill, in fact all of the senators favor the bill.
- It is not the case that the Holy One answers all prayers, in fact the Holy One answers no prayers.
It is not the case I have just listened to some symphonies by Shostakovich, I have in fact listened to all of them.

Logical negation, on the other hand, is designed to close all loopholes. So suppose in court a lawyer asks a witness: 'is it true that you have not stolen a lot of money from your company?' and the witness says 'yes'. Then suppose that we later find out that the witness literally stole all of the company's money. The witness then claims to have answered truthfully because 'a lot' implies roughly 75% whereas they stole 100%. Further, 75% and 100% are not the same, hence it is consistent to not steal 75% and steal 100% at the same time. In a logically perfect language logical and natural negation are disambiguated. 'Not' is used for logical negation and 'notn' stands for natural negation where the 'n' means natural. Hence, if you assert 'not manyb senators favor the bill' then four other possibilities follow: 'all senators favor the bill, a few senators favor the bill, one senator favors the bill or no senators favor the bill.'

\[ 194. (\text{not many}_n b \ R c) \equiv ((\text{few } b \ R c) \ \lor \ (\text{every } b \ R c) \ \lor \ (a \ b \ R c) \ \lor \ (\text{no } b \ R c)) \]
\[ 195. (\text{notd many}_n b \ R c) \equiv (\text{few } b \ R c) \]

6.4.8. The Indicative Inference and Subjunctive Inference

Before we define the conditional quantifiers we need to talk about the subjunctive and the indicative tense which is currently collapsing in popular English. In our logic, however, we cannot confuse these two but must always make it unambiguous whether the relationship we assert is indicative or subjunctive which is to say whether it is true in the actual world or in a non-actual world. So in our logic when we say:

196. g1 Every tachyon travels faster than the speed of light.

we imply that there are tachyons in reality, not just in the imagination. Because there are no tachyons in reality we must use the subjunctive tense:

197. g2 Every tachyon would travel faster than the speed of light.

For this reason, in our logic when we use the word 'every' and the indicative tense as in the sentence 'every Beatle is British' we must infer that there are Beatles in reality. Sometimes we simply write this inference into our definitions since this rule is somewhat hard to code for. But eventually we would like to make it so that this inference is only made if it is needed to argue for a claim. Right now this rule is called the 'Axiom of Definition', but in the future we might change it to: 'The Axiom of the Indicative Tense'. Similarly, with the subjunctive tense, when we say 'every tachyon would travel faster than the speed of light' we must make the inference that there are no tachyons in reality.
6.4.9. The Definition of the Conditional Quantifiers 'Every', 'No' and 'Many'

A conditional quantifier is a determiner which contains either → or ≡ in its definiens. So far we have only found three of these 'every', 'no' and 'many'. The determiners 'only' also has this property but we're not going to investigate 'only'. The universal quantifiers are 'every' and 'no'. 'All' is synonymous with 'every' but modifies a plural noun but since plurals and singulars are indistinguishable in this logic we will focus exclusively on 'every'. We have not done any research on whether or not 'every' and 'each' are synonymous.

The definition of these for the very simple cases is basically the same as it is in basic predicate logic except that whereas in predicate logic the logician has to manually translate a sentence with 'every' in it into notation, in natural language logic, this is done automatically. For example, in

198. No man is mortal.

The logician is supposed to understand that the translation of that into predicate logic is:

\((\forall x)((Mx) \rightarrow (\neg MOx))\)

It is not always obvious however how a natural language sentence is to be translated into predicate logic and hence disagreement can arise. For example, is the sentence 'every man who owns a donkey beats it' translated as:

\((\forall x) (\forall y)((Mx & Dy & Oxy) \rightarrow Bxy)\)

or because the indefinite article precedes 'donkey' is it translated as:

\((\forall x) (\exists y)((Mx & Dy & Oxy) \rightarrow Bxy)\)

In NLL 'no' in 198 is defined as:

\(((\text{no } b \ J \ c) \equiv ((d \ I \ b) \rightarrow (d \sim \ J c)) & (b=\text{man}) & (c=\text{mortal})\)

The same applies with the definition of 'every'. Things become much more complicated when we deal with sentences with subordinate clauses, adjectives and prepositional relations.

**Definition: Singular Indefinite Clauses and Plural Indefinite Clauses**

b is a singular indefinite clause iff b is a clause and b modifies c and d is modified by a singular indefinite article.

b is a plural indefinite clause iff b is a clause and b modifies c and d is modified by a plural indefinite article.
199. A dog that had brown hair went into every home on 4th street.
200. Ah smartphone built by Apple was awarded to every engineer.

In 199 the clause 'that had brown hair' modifies 'dog' which is modified by the singular indefinite article since it is one dog that went into every home. In 200 the clause 'built by Apple' modifies 'smartphone' which is modified by the plural indefinite article since each engineer will receive a different smartphone.

**Definition: Plural Indefinite Adjective**

b is a plural indefinite adjective iff b modifies c and d modifies c and d is a plural indefinite article.

201. A black dog went into every home on 4th street.
202. Ah black phone was awarded to every engineer.

In 202 black modifies a noun which is modified by the plural indefinite and hence it is a plural indefinite adjective. In 201 because 'dog' is modified by the singular indefinite article 'black' is not a plural indefinite adjective.

**6.4.10. Decision Procedure for Words Under Scope**

*Words which come under the scope of a conditional quantifier must be defined at the same time as the conditional quantifier.*

**6.4.11. Definition: Come under the Scope of a Conditional Quantifier**

1. The subordinate relation whose relata are clauses is a transitive relation, that is to say, if clause b is subordinate to clause d and clause d is subordinate to clause e, then b is subordinate to e.

2. If a word exists in the same clause as a conditional quantifier then it may come under the scope of that conditional quantifier.

3. If a noun b is modified by a conditional quantifier c and a clause d modifies b then each word in d may come under the scope of c.

4. If a clause b comes under the scope of c and d is subordinate to b then d comes under the scope of c.
5. If any of the following types of words are the plural indefinite article or are modified by the plural indefinite article and they meet conditions 1 or 2 then they come under the scope of the conditional quantifier:

any\textsuperscript{a}, the\textsuperscript{i}, nouns, adjectives, common name possessives, relative pronouns, prepositional relations.

203. Every man [who was] in the van [which was] by the bridge wore a\textsuperscript{h} black shirt.

In 203 although each 'the' meets condition 2 it does not meet condition 5. The indefinite article, on the other hand, does meet both conditions 2 and 5 and thus comes under the scope of 'every' and hence is not defined until 'every' is defined.

204. I sang a\textsuperscript{h} song during every minute of the first hour of a\textsuperscript{h} fine day in the spring.

In 204 I think it is ambiguous whether or not a\textsuperscript{h} modifies 'day' or 'a' modifies 'day' though 'a' is the more likely reading, so we'll just see what happens with either one.

Because the first plural indefinite article exists in the same clause as 'every' it comes under the scope of 'every' and hence cannot be defined until 'every'. Because 'first' is not an indefinite plural adjective and because 'the' never comes under the scope of the conditional quantifier, it can be defined in its normal order which is before the conditional quantifier. The 'a\textsuperscript{h}' which modifies 'day' on the other hand belongs to the same clause as 'every' and thus comes under the scope of 'every' and hence it and 'fine' are defined at the same time as 'every'.

Let's now see what happens if 'a' modified 'day' rather than 'a\textsuperscript{h}'. We have not yet actually placed this in our definition yet but 'a' entails 'exactly one' whereas a\textsuperscript{a}, a\textsuperscript{h} and a\textsuperscript{d} do not. Because 'a' does not come under the scope of 'every', it will be defined before 'every' and will thus become an indefinite object. It will then be possible that if anything is a fine song that it is identical to that fine song I sang in spring but it will not be possible that two different songs are identical to that fine song.

205. I saw a man who had no money deposited in any\textsuperscript{a} bank, walk to a bar.

In 205 the first indefinite article belongs to a clause superordinate to 'no' and hence does not come under its scope but even if it did it would still not come under its scope since it is the singular 'a'. The determiner 'any\textsuperscript{h}', however, is both within the same clause as 'no' and belongs to the list of words found in 5 and hence comes under its scope.

206. Every person sleeping in that cabin had seen a\textsuperscript{h} man who had the\textsuperscript{a} bit between his\textsuperscript{a} teeth.

Because 'that' meets condition 2 but not condition 5 it does not come under the scope of 'every'. Because 'who had the bit between his teeth' is an indefinite sub-clause it thus
comes under the scope of 'every'. If instead of 'a\(^h\)\) man' we had 'a man', the sub-clause would be definite and it would not come under the scope of 'every'.

207. I sang ah song during many of the days in winter.

Because a\(^h\) exists in the same clause as many\(^a\) and because many\(^a\) is a conditional quantifier, it comes under its scope.

6.4.12. Quantification Rules

Definition: Instance of the Quantified Noun

'Every man is mortal' is equivalent to 'if b is a man then b is mortal'. In the latter sentence b is the instance of the quantified noun.

Quantification Rule 1

Change the conditional quantifier to the indefinite article and subscript the quantified noun with same letter since this noun will now appear twice. This is only a pedagogical device and is not used in our formal arguments.

Quantification Rule 2

If a sentence begins with a universal quantifier and the first relation is not a main relation then each word before the main relation will become part of the antecedent, the instance of the quantified noun will become the subject of the main relation and each word thereafter will become part of the consequent.

Quantification Rule 3

If the first relation is the main relation and a universal quantifier appears after the first relation and before a prepositional relation, then every word after the conditional quantifier beginning with the now transformed conditional quantifier becomes the antecedent and the consequent is composed of every word from the beginning of the sentence until the conditional quantifier. Further, the instance of the quantified noun will be the last word of the consequent.

Quantification Rule 4

If a sentence ends with a noun universally quantified or begins with a noun universally quantified followed immediately by the main relation, then the antecedent is composed simply of the quantified noun preceded by the indefinite article.

Quantification Rule 5
When an antecedent $b$ or consequent $b$ is reduced to a sentence $c$ in standard form then $c$ directly replaces $b$, skipping over all intermediary steps.

208. Every man born on Mars loves a$^h$ woman born on Venus.
209. ($b$=man) & ($c$=mars) & ($d$=woman) & ($e$=venus)
210. (every $b$ BRN $c$ LO $a$ d BRN $e$)

211. ($a$ ba BRN ON $c$) → ($ba$ LO $a^h$ d BRN ON $e$)

The only thing we need to do to reduce 88 to standard form is eliminate the indefinite articles. We now instantiate $b$ with $f$ and $d$ with $g$. $f$ will become the instance of the quantified noun which then becomes the subject of the LO relation. The important point is that we do not do things in the following way:

212. ($a$ ba BRN ON $c$) ≡ (($f$ BRNON $c$) & ($f$I $b$)) DF $a$

Because we instantiated $b_a$ with $f$ we must do that wherever we see $b_a$:

213. ($ba$ LO $ah$ d BRN ON $e$) ≡ (($f$ LO $a^h$ d BRN ON $e$) & ($f$I $b$)) DF $a$
214. ($f$ LO $ah$ d BRN ON $e$) ≡ (($f$ LO $g$ BRN ON $e$) & ($g$I $d$)) DF $a^h$

That's too hard to calculate with and too confusing. Instead in our argument we do not write steps 212 - 214 but simply write:

(every $b$ BRN $c$ LO $a$ d BRN ON $e$) ≡ (((($f$ BRN ON $c$) & ($f$I $b$)) → (($f$ LO $g$ BRN ON $e$) & ($g$I $d$)))

It is also important to recall instantiation rule 6:

If a sentence is detached from a hypothetical sentence which contains an abbreviation which was once modified by $a^h$ then each time it is detached the indefinite abbreviation must be translated into a new abbreviation.

So suppose 'Richard and Brad are men born on Mars and Elizabeth and Angelina are women born on Venus. It does not follow that Richard loves either Elizabeth or Angelina, nor does the same follow for Brad. When we instantiate 'man' with Richard we are obligated to change the indefinite variable, here $g$, to something else, say $k$. So if Elizabeth is abbreviated with $e$, then it will only follow contingently that $e$ is identical to $k$.

6.4.13. Using Quantification Rule 2

215. Every man in the van by the bridge wore a$^h$ black shirt.
216. ($b$=man) & ($c$=van) & ($d$=black) & ($e$=shirt) & ($f$=definite) & ($g$=bridge)
217. (every man in the van by the bridge wore a\textsuperscript{h} black shirt) \equiv (every b IN the c BY the g WER a\textsuperscript{h} d e)

Because the word 'the' never comes under the scope of a conditional quantifier, it can be defined before 'every'. We will define both definite articles at once:

218. (every b IN the c BY the g WER a\textsuperscript{h} d e) \equiv ((every b IN h BY g WER a\textsuperscript{h} d e) & (h I c) & (j I g) & (j.h J f))

To reduce (every b IN f BY g WER a\textsuperscript{h} d e) we note that the main relation is WER, hence the dividing line between antecedent and consequent is (every b IN f BY g \ WER a\textsuperscript{h} d e). After we change 'every' to 'a' we will have:

219. ((a ba IN h BY j) \to (ba WER a\textsuperscript{h} d e))

We will first instantiate b with k:

220. (((k IN h BY j) & (k I b)) \to (k WER a\textsuperscript{h} d e))

In the sentence (k WER a d e) we first instantiate e with an arbitrary abbreviation, here m:

221. (k WER ah d e) \equiv ((k WER d m) & (m I e))

We now eliminate the adjective d:

222. (k WER d m) \equiv ((k WER m) & (m J d))

In our argument, steps 219 to 222 are skipped and we simply write:

223. (every b IN f BY g WER ad e) \equiv
((((k I b) & (k IN f BY g)) \to (k WER m) & (m J d) & (m I e))) \quad DF every

6.4.14. Using Quantification Rule 3

224. I sang ah song during every minute of the first hour of the day.
225. (b=song) & (c=minute) & (d=hour) & (e=day) & (f=first) & (g=definite)
226. (i SNG ah b DUR every c OF the f d OF the e)

We first define 'the' since 'the' never comes under the scope of a conditional quantifier:

227. (i SNG ah b DUR every cOF f h OF k) & (k I e) & (h I d) & (k.h J g)
Because 'first' modifies a definite noun, we must use predicative complement insertion on f:

228. \((i \text{ SNG } a \ b \ \text{DUR} \ \text{every } c \text{OF } f \ h \ \text{OF } k) \equiv ((i \text{ SNG } a^b \ b \ \text{DUR} \ \text{every } c \ \text{OF } h \ \text{OF } k) \& \ (h \ J \ f))\) 

Because 'every' appears after the main relation we thus draw the line between antecedent and consequent where the \ is in the following sentence, except that what comes before the \ goes into the consequent:

229. \((i \text{ SNG } a \ b \ \text{DUR } \backslash \text{every } c \ \text{OF } h \ \text{OF } k)\)

230. \((a \ ca \ \text{OF } h \ \text{OF } k) \rightarrow (i \text{ SNG } a^h \ b \ \text{DUR } c_a)\)

We now eliminate 'a' in 230:

231. \((a \ ca \ \text{OF } h \ \text{OF } k) \equiv ((m \ \text{OF } h \ \text{OF } k) \& \ (m \ I \ c))\) 

232. \((m \ \text{OF } f \ h \ \text{OF } k) \equiv ((m \ \text{OF } h \ \text{OF } k) \& \ (h \ J \ f))\)

In 230 we add the instance of the quantified noun:

233. \((i \text{ SNG } a \ b \ \text{DUR } m)\)

We now eliminate 'a' in 233:

234. \((i \text{ SNG } a \ b \ \text{DUR } m) \equiv ((i \text{ SNG } n \ \text{DUR } m) \& \ (n \ I \ b))\)

Hence, our full definition of 'every' in 227 is:

235. \((i \text{ SNG } a \ b \ \text{DUR} \ \text{every } c \ \text{OF } f \ h \ \text{OF } k) \equiv ((m \ \text{OF } h \ \text{OF } k) \& \ (m \ J \ f) \& \ (m \ I \ c)) \rightarrow ((i \text{ SNG } n \ \text{DUR } m) \& \ (n \ I \ b)))\) 

6.4.15. Using Quantification Rule 4

Quantification rule 4 is only used when the conditional quantifier modifies the last noun or the first noun in a sentence where the first relation is the main relation.

236. I sang a song during the first hour of every day.
237. \((b=\text{song}) \& \ (c=\text{first}) \& \ (d=\text{hour}) \& \ (e=\text{day}) \& \ (g=\text{definite})\)
238. \((i \text{ SNG } ah \ b \ \text{DUR } f \ \text{OF } \text{every } e) \& \ (f \ I \ d) \& \ (f \ J \ g) \& \ (f \ J \ c)\)

In 238 'the' has already been defined. Note that because \(a^h\) comes under the scope of 'every' so long as it exists in the same clause as 'every', it is not defined now.
239. \((a \text{ ea}) \rightarrow (i \text{ SNG ah b DUR f OF } e_a)\)

Recall that even though \((a \text{ ea})\) is not well-formed this does not matter because that sentence will not appear in our argument but is a mere pedagogical device. We now instantiate \(e\) with \(h\):

240. \((h \text{ I e}) \rightarrow (i \text{ SNG ah b DUR f OF } h)\)

We now define \('ah'\) in 240:

241. \((i \text{ SNG ah b DUR f OF } h) \equiv ((i \text{ SNG k DUR f OF } h) \& (k \text{ I b}))\)  
DF \(a^h\)

Hence, the definition of 'every' in 238 is:

242. \((h \text{ I e}) \rightarrow ((i \text{ SNG k DUR f OF } h) \& (k \text{ I b}))\)

For those sentences that begin with 'every' and the first relation is the main relation:

243. Every boy plays baseball in the morning during the summer.

We're not now going to get into pragmatics but just to illustrate what awaits us in the future, 243 is pragmatic for:

244. Every boy plays ah game of baseball in the morning during the summer.

It might even be the case that 244 is still not right but what is meant is:

245. Every boy plays ah game of baseball every morning during each summer of his boyhood.

But we won't get into those details now. As usual, 244 is false but it illustrates a point:

246. \((b=\text{boy}) \& (c=\text{game}) \& (d=\text{baseball}) \& (e=\text{morning}) \& (f=\text{summer}) \& (g=\text{definite})\)

247. \((\text{every b PL ah c OF d IN the e DUR the f}) \equiv ((\text{every b PL a}^h \text{ c OF d IN h DUR j}) \& (h \text{ I e}) \& (j \text{ I f}) \& (h.j \text{ J g}))\)

248. \((a \text{ ba}) \rightarrow (ba \text{ PL a c OF d IN h DUR j})\)

249. \((a \text{ ba}) \equiv (k \text{ I b})\)

250. \((k \text{ PL ah c OF d IN h DUR j}) \equiv ((k \text{ PL m OF d IN h DUR j}) \& (m \text{ I c}))\)

Hence:

251. \((\text{every b PL ah c OF d IN h DUR j}) \equiv ((k \text{ I b}) \rightarrow ((k \text{ PL m OF d IN h DUR j}) \& (m \text{ I c})))\)  
DF every
6.4.16. Formalizing Donkey Sentences

Let's now show how donkey sentences are formalized. They are essentially formalized just the same as 'a' or 'a^h' except that the noun modified by a^f is referred to once in the same clause as 'every' and once again in a clause subordinate to 'every'. Because they are referred to twice, they automatically acquire the properties of full variables.

252. Every girl who sees a handsome prince falls in love with him.

In formalizing 252 we decided to make 'in love' redundant and all the meaning captured in 'fall in love' is contained in the word 'fall':

253. (b=girl) & (c=handsome) & (d=prince)
254. (every b who SEE af c daFL d)

Our first step is to separate the antecedent from the consequent:

255. (a bb who SEE af c d) \rightarrow (b FL d)

We now instantiate b with e and get:

256. ((e who SEE af c da) & (e I b)) \rightarrow (e FL d)

Now that we know that b is instantiated with e we can eliminate 'who':

257. (e who SEE af c da) ≡ (e SEE af c da)

We thus instantiate d with f and get:

258. (e SEE af c da) ≡ ((e SEE c f) & (f I d))

We now eliminate the adjective c as follows:

259. (e SEE c f) ≡ ((e SEE f) & (f J c))

Now that we have shown what 254 is equivalent to, we must combine all of the steps from 255 to 259 together into one:

260. (every b who SEE af c d FL d) ≡ (((e SEE f) & (f J c) & (e I b) & (f I d)) \rightarrow (e FL f))
So let's suppose that the girl Kate sees Prince Leopold but he is not handsome. It will not follow from our rules that Kate will fall in love with him. This is because Leopold must have all of the antecedent properties that f has in 260 which are three: 'be seen by Kate', 'be a prince' and 'be handsome'. Leopold only has two of those properties. However, suppose William has all of those properties. It will then follow that Kate falls in love with him.

Let's now suppose that Kate will not fall in love with any handsome prince she sets her eyes on but only a particular prince whose identity is unknown to us. This would be formalized as:

261. There is a handsome prince and if Kate sees him, then she will fall in love with him.

Even though 'every' is not found in 261 it is important to distinguish between 'a' and 'a^f' which will be illustrated after we reduce it to standard form.

262. (b=kate) & (c=handsome) & (d=prince)
263. (there EX a c da) & ((b SEE him_a) → (b FL him_a))
264. (there EX a c da) ≡ ((there EX c e) & (e I d))  DF a
265. (there EX c e) ≡ ((there EX e) & (e J c))  PCI
266. (there EX e) ≡ (e EX)

Now that we know that 'prince' is instantiated with e, we can replace the words which are subscripted with e:

267. ((b SEE him_a) → (b FL him_a)) ≡ ((b SEE e) → (b FL e))

Our standard sentences are now:

268. (b SEE e) → (b FL e)
269. (e EX) & (e J c) & (e I d)

Recall that the listener does not know who the handsome prince is and the speaker may or may not know his identity. It is not clear from the words one way or the other. Now, suppose Leopold is a handsome prince and Kate sees him. Intuitively we understand that it is only possible that Kate will fall in love with Leopold since we do not know the identity of the handsome prince that will charm Kate if seen. This is exactly what our rules predict because in 269 e is an indefinite abbreviation and therefore any other detached abbreviation whether constant or indefinite can be identical to e so long as they do not share contradictory properties.

We now want to make sure that our definition of donkey articles can make the following inferences:
270. Every man who beats a donkey is abusive. Aristotle beat a donkey. Therefore, Aristotle is abusive.
271. Aristotle beat a donkey. Russell owns a donkey. Therefore it is possible that Aristotle beat Russell's donkey.

Admittedly, 270 is false because you have to beat a donkey several times before you are considered to be abusive but that is beside the point:

272. (b=man) & (c=donkey) & (d=aristotle) & (e=abusive) & (f=russell)
273. (every b who BET ah c J e) ≡ (((g I b) & (h I c) & (g BET h)) → (g J e))

In 271 the indefinite article modifies a noun which will be represented with an indefinite variable like so:

274. (d BET a c) ≡ ((d BET j) & (j I c))

When we detach the biconditional consequent from the biconditional antecedent in 273 the h abbreviation will be found within the antecedent of a conditional and not in the consequent, nor will it be found in a detached sentence and hence it is a half general variable. That is to say, that any abbreviation which has all of the properties of h MUST instantiate h. The indefinite abbreviation j in sentence 274, on the other hand, after it is detached will exist within a detached sentence and not in a hypothetical sentence and hence it will be indefinite. That is to say that if m does not share contradictory properties with j, then it is contingent that m is intrinsically identical to m.

The main inference we want to block here is that if Aristotle beats a donkey it does not follow that he beats every donkey, it only follows that he is abusive. So the conclusion we want to obtain is

35e ((k I c) → (d BET k))

since 275 is reduced as follows:

275. (aristotle beat every donkey) ≡ (d BET every c)
276. (d BET every c) ≡ ((k I c) → (d BET k))

After we use detachment on 273 - 276 we will have the standard sentences:

277. ((g I b) & (h I c) & (g BET h)) → (g J e)
278. (d BET j) & (j I c) & (dl b)

Using 277 and 278 we cannot obtain the conclusion 35e but can only obtain (g J e). Hence, it does not follow from 'every man who beats a donkey is abusive and Aristotle beat a donkey' that 'Aristotle beat every donkey'.
6.4.17. The Biconditional 'Every'

In very rare circumstances the $\equiv$ sign is found in the definition of 'every' rather than $\rightarrow$. When this happens we superscript 'every' with 'b' which stands for biconditional. For example,

279. Everyb triangle has threesides.

The correct reduction of 279 would be:

280. (b=triangle) & (c=three) & (d=side)
281. (everyb b c d) $\equiv$ ((e I b) $\equiv$ (e H c d))

The reduction of (e H c d) is:

282. (e H c d) $\equiv$ ((e H g) & ((g W h) $\rightarrow$ (h I d)) & (g N c))

*e has three sides iff e has [set] g and if g has $^o$ h then h is $^o$ [a] side and g instantiates 3.*

So the definition of (every$b$ b H c d) would be:

283. (everyb b H c d) $\equiv$ ((e I b) $\equiv$ ((e H g) & ((g W h) $\rightarrow$ (h I d)) & (g N c)))

**Definition: To Anaphorize a Noun or Adjective**

It is very hard to explain what anaphorize a noun means but it means roughly you take a word which would appear in the consequent of the definiens if the quantifier were 'every' but because the quantifier is 'no' you put it in the antecedent of the definiens. For example, suppose you have the artificial sentence which you know will be divided into an antecedent and a consequent:

284. No Helot spoke to a Spartan.
   (b=helot) & (c=spartan)
285. (no b SPK af c)

Because the abbreviation 'c' in 285 will eventually be instantiated with a full-general variable we must superscript 'a' with 'f'. If we instantiate b with d then we will have the following sentence:

286. (d I b) $\rightarrow$ (d $\sim$ SPK af c)

We anaphorize c by instantiating it with an arbitrary abbreviation, say e. We then place e in the object position of the SPK relation and we also place the sentence (e I c) in the antecedent:

287. ((d I b) & (e I c)) $\rightarrow$ (d $\sim$ SPK e)
This is called anaphorizing the noun c. If the noun is modified by an adjective then it is anaphorized as follows:

288. No woman spoke to a red donkey.
289. (b=woman) & (c=red) & (d=donkey)
290. (no b SPK af c d)

We then take the not well-formed sentence (a c d) and reduce it as we normally would: (f I d) & (f J c). We then place f in the object position of the SPK relation and we place (f I d) & (f J c) in the antecedent, like so:

291. (no b SPK af c d) ≡ ((e I b) & (f I d) & (f J c)) → (e ~ SPK f)

If the indefinite noun should be modified by a relative pronoun then it is anaphorized as follows:

292. No woman spoke to a donkey which was owned by Ibn Sina.
293. (b=woman) & (c=ibn sina) & (d=donkey)
294. (no b SPK af d which OWNB c)

We then take the indefinite noun and all those words that modify it which are (a d which OWNB c) and reduce that sentence like we would any other, then place it in the antecedent.

294. (no b SPK af d which OWNB c) ≡ (((e I b) & (f I d) & (f OWNB c)) → (e ~ SPK f))

6.4.18. Quantification Rule 6

Words which come under the scope of 'no' are anaphorized.

6.4.19. Quantification Rule 7

If the consequent within the definiens is negated and the consequent happens to be a conjunction then the entire conjunction is negated. For instance, if the conjunction were (p & q), then it would be negated like ~(p & q), not (~p & q).

296. No woman spoke to a man sitting in a black car on 3rd street at high noon.

Because 'man' is indefinite and because 'sitting in a black car on 3rd street at high noon' modifies 'man', that whole phrase must be anaphorized.
297. (b=woman) & (c=man) & (d=black) & (e=car) & (f=3rd street) & (g=high noon) 
298. (no b SPK af c SIT af d e ON f T g)

Let's first reduce the anaphorized noun by defining 'af' and performing predicative complement insertion:

299. (af c SIT af d e ON f AT g) ≡ ((h I c) & (j I e) & (j J d) & (h SIT j ON f T g))

After we instantiate b with k, we get:

300. ((k I b) & (h I c) & (j I e) & (j J d) & (h SIT j ON f T g)) → (k ~ SPK h)

Let's now suppose that Aristotle is a man who sat in a red car on 3rd street at high noon. Would it be a contradiction if a woman spoke to him? Let's find out:

301. Aristotle isg ar man whosat in a red car on 3rd street at' high noon.
302. (m=aristotle) & (n=red) & (b=woman) & (c=man) & (d=black) & (e=car) & (f=3rd street) & (g=high noon)
303. (m I c who SIT a n e ON f T g)

We first define 'a' and perform predicative complement insertion and we will combine them into one step, though normally we do not do that:

304. (m I c who SIT a n e ON f T g) ≡ 
   ((m I c who SIT o ON f T g) & (o I e) & (o J n)) DF a, PRI

We now define 'who':

305. ((m I c who SIT o ON f T g) ≡ ((m I c) & (m SIT o ON f T g)) DF who

Hence, in order for Aristotle to be instantiated by c in

((k I b) & (h I c) & (h SIT j) & (j I e) & (j J d) & (j ON f T g)) → (k ~ SPK h)

he must have the following properties: (αIc) and (α SIT γ ON f T g). Aristotle has those properties but now o must have the property 'being black' and it does not. Thus, we will never be able to detach the consequent and infer that no woman spoke to Aristotle.

Let's now do an example where the object is definite but later on in the sentence there is an indefinite object.

306. No woman during the party spoke with the man who wore a green coat.

One of the challenges to 306 is that we have to program the computer to get the prepositional phrase 'during the party' in the right place. We haven't worked on this problem yet so for now it will just be moved to the end and we will just assume that our
program is written such that 'during the party' is not part of the subordinate clause 'who wore a green coat'.

307. No woman spoke with the man who wore a\textsuperscript{f} green coat during the party.
308. (b=woman) & (c=man) & (d=green) & (e=coat) & (f=party) & (g=definite)
309. (no b SPK the c who WER af d e DUR the f) 
309. (no b SPK the c who WER af d e DUR the f) ≡

((no b SPK h who WER a\textsuperscript{f} d e DUR j) & (h I c) & (j I f))

Because 'who' and 'a\textsuperscript{f}' and the adjective d come under the scope of 'no' they must all be defined at once. Let's first concentrate on the anaphorized noun 'e':

\[(a^f d e) ≡ ((m I e) & (m J d))\]

Hence, we now have the following:

\[((a b_a) & (m I e) & (m J d)) \rightarrow (b_a SPK h who WER m DUR j)\]

We now define 'a' and eliminate 'who':

\[((n I b) & (m I e) & (m J d)) \rightarrow ((n SPK h) & (h WER m DUR j))\]

Because 'no' has a negation in its definiens we must negate the whole consequent not just part of it:

\[75e ((n I b) & (m I e) & (m J d)) \rightarrow \neg((n SPK h) & (h WER m DUR j))\]

Let's now suppose there was a different man at the party who wore a red coat and all we know about him is that he is different from h. So let's refer to him with o. Would it be a contradiction if a woman spoke to him? No, for two reasons. Number one, h is a constant and hence it cannot be instantiated with o. Second, let's suppose h was not a constant, it still would not be a contradiction if a woman spoke to him because he wore a green coat, whereas o wore a red coat and we can only get a contradiction if a woman speaks to the man wearing the red coat.

Let's now do an example where 'no' appears after the main relation.

311. A man who lived on the 4th floor listened to Mozart at\textsuperscript{f} no hour of the day which occurred before Good Friday.
312. (b=man) & (c=4th) & (d=floor) & (e=Mozart) & (f=hour) & (g=day) & (h=definite) & (r=good friday)
313. (a b who LV ON the c d LST e T no f OF the g which OC BF r)

Note that the words 'a' and 'who' and the rule predicative complement insertion do not come under the scope of the conditional quantifier. This is because 'a' does not come under the scope of 'no' but only 'a\textsuperscript{h}' and 'a\textsuperscript{f}' do. Further, since 'man' is modified by 'a' it follows that the clause introduced by 'who' also does not come under the scope of 'no'.

115
We can thus define 'a', 'who' and use predicative complement insertion like we normally would:

313. \[(a \ b \ who \ LV \ ON \ the \ c \ d \ LST \ e \ T \ no \ f \ OF \ the \ g \ which \ OC \ BF \ r) \equiv (a \ b \ who \ LV \ ON \ the \ c \ d \ LST \ e \ T \ no \ f \ OF \ the \ g \ which \ OC \ BF \ r) \& (k \ I \ b)\]  
\(\≡\)  
DF a

315. \[(k \ who \ LV \ ON \ the \ c \ d \ LST \ e \ T \ no \ f \ OF \ the \ g \ which \ OC \ BF \ r) \equiv ((k \ who \ LV \ ON \ c \ m \ LST \ e \ T \ no \ f \ OF \ n) \& (m \ I \ d) \& (n \ I \ g) \& (m \ n \ J \ h))\]  
\(\equiv\)  
DF the

316. \[(k \ who \ LV \ ON \ c \ m \ LST \ e \ T \ nof \ OF \ n \ which \ OC \ BF \ r) \equiv ((k \ who \ LV \ ON \ c \ m \ LST \ e \ T \ no \ f \ OF \ n \ which \ OC \ BF \ r) \& (k \ LST \ e \ T \ no \ f \ OF \ n) \& (k \ LV \ ON \ c \ m))\]  
\(\equiv\)  
DF who

317. \[(k \ LV \ ON \ c \ m) \equiv ((k \ LV \ ON \ cm) \& (m \ J \ c))\]  
PCI

Our only task now is to define 'no' in the following:

318. \[(k \ LST \ e \ T \ \no \ f \ OF \ n \ which \ OC \ BF \ r)\]

Because 'no' appears in one of the prepositional relations to the right of the main relation every word to the left of the \ will appear in the consequent and everything on the right will appear in the antecedent, like so:

\[(a \ f \ a \ OF \ n \ which \ OC \ BF \ r) \rightarrow (k \ ~ \ LST \ e \ T \ f_\alpha)\]

We now reduce \((a \ f_\alpha \ OF \ n \ which \ OC \ BF \ r)\) like we would any normal sentence:

\[(a \ f_\alpha \ OF \ n \ which \ OC \ BF \ r) \equiv ((o \ I \ f) \& (o \ OF \ n \ which \ OC \ BF \ r))\]  
\(\equiv\)  
DF a

\[(o \ OF \ n \ which \ OC \ BF \ r) \equiv ((o \ OF \ n) \& (n \ OC \ BF \ r))\]  
\(\equiv\)  
DF which

Hence, the definition of 318 is:

\[(k \ LST \ e \ T \ no \ f \ OF \ n \ which \ OC \ BF \ r) \equiv ((o \ I \ f) \& (o \ OF \ n) \& (n \ OC \ BF \ r)) \rightarrow (k \ ~ \ LST \ e \ T \ o))\]
6.4.20. The Definition of 'Many'

many

This is the 'many' which is atomic and which is a predicative complement. This must be a property of a group, hence it is always the object of the J relation. Its natural language correspondent is the highly awkward:

319. This group is a many.

It is very important to point out that the negation of this meaning of 'many' does not mean 'none' but 'few'. Because 'this group is a not many' is too awkward, we simply write \((b \sim J c) \& (c=\text{many})\). From \((b \sim J c)\), it still follows that \(b\) instantiates a number greater than one, but that is a property of all groups.

many\(^b\)

320. More than one. E.g. True love is one not many.
321. \((b=\text{many}^b) \& ((c J b) \equiv ((c N d) \& ((d=e) \lor (d G e)))) \& (e=2)\)

If \(c\) is \(a\) many\(^b\) then \(c\) instantiates a number greater than or equal to 2.

The negation of 'many\(^b\)' is one. This is invariably found in expressions such as 'France wanted Germany to be many not one.' Further, this many\(^b\) is a predicative complement.

many\(^a\)

322. Many senators favor the bill iff the majority of senators favor the bill.

This is the main meaning of 'many' that we are interested in. When I use this many\(^a\) I use it as a determiner and imply that not all senators favor the bill. What this means is that all of the American senators are divided into two sets: those that favor the bill and those that do not. Moreover, I also have an additional belief that the set of those that favor the bill are larger than those that do not.

So the first set is formalized as follows:

\((d=\text{senator}) \& (e=\text{bill})\)
\((b W c) \equiv ((c I d) \& (c FV e))\)

\(b\) has \(w\) \(c\) iff \(c\) is \([a]\) \(d\) and \(c\) favors \(e\)

The second set is formalized as:
\[(f \ W \ c) \equiv ((c \ I \ d) \ & \ (c \sim \ FV \ e))\]

We then state that set b is larger than set f:

\[(b \ GR \ f)\]

which, by the way, has the following equivalence:

\[(b \ GR \ f) \equiv ((d \ N \ h) \ & \ (f \ N \ j) \ & \ (h \ G \ j))\]

We then use the Axiom of Indicative Tense:

\[(j \ . \ k \ I \ d) \ & \ (j \ FV \ e) \ & \ (k \sim \ FV \ e)\]

We would rather only use that axiom if it is strictly necessary but at the moment it is somewhat hard to program the computer to make that inference only when needed. A preliminary definition of 'many' would be:

\[(many^n_b \ R \ c) \equiv (((d \ W \ e) \equiv ((e \ R \ c) \ & \ (e \ I \ b))) \ & \ ((f \ W \ g) \equiv ((g \sim \ R \ c) \ & \ (g \ I \ b))) \ & \ (d \ GR \ f) \ & \ (m \ R \ c) \ & \ (m \ I \ b) \ & \ (o \sim \ R \ c) \ & \ (o \ I \ b))\]

The above is preliminary because we also need to state how we incorporate the other parts of the sentence that it is found in into the definition.

\[many^d\]

There were many cars in the parking lot iff there were more cars in the parking lot than usual.

Because the meaning of 'usual' is quite difficult we're not going to do any work on this determiner at the moment, even though when we use the word 'many' this is the meaning most often meant. The best way to distinguish 'many' from 'many' is to simply replace 'many' with the majority of and see if a contradiction arises. In 'the mother has many kids in her car', because 'the mother has the majority of kids in her car' is false, it therefore follows that 'many' is meant. And what this sentence means is that usually a mother has no more than two kids in her car. So if a mother has three kids in her car it would be correct to say that she has many kids in her car. On the other hand, if I were to say 'I've read many philosophy books by Russell,' it would be correct to infer in this case that I've read the majority of Russell's philosophy books. Still, if one were to say 'I've read many of Goethe's writings,' since his collected works are 144 volumes, what the speaker of that sentence means is that most Germans (for anglophones the number is different) who read Goethe read the equivalent of one or two of those volumes, but this speaker has read at
least four of these volumes or whatever number one thinks demarcates the usual from the rare.

**many**

323. Consistent with 'all', more than one, e.g. If the Godfather has all of the politicians in his pocket then the Godfather has many politicians in his pocket.
324. In 'James Joyce has a dog', 'a' refers to one object, whereas in 'every man has a dog', 'a' refers to many objects.

This is the determiner counterpart of 'many'. Notice that in 324 I'm not putting the objects that the second indefinite article refers to into a group and making a claim about the size of the group. I'm just stating that these objects are plural and not singular. Further, it is consistent to assert that "If the Godfather has all of the politicians in his pocket then the Godfather has many politicians in his pocket." When we use this meaning of 'many' we do not imply that there is a politician which the Godfather does not have in his pocket.

### 6.4.21. Using the Quantification Rules to Define 'Manyn'

The quantification rules 2 - 5 are used when defining 'many'.

325. Manyn men in the van by the bridge wore a black shirt.

We first transfer the prepositional relations to the end:

326. Manyn men wore a black shirt in the van by the bridge.
327. 

Because 'a' in the following comes under the scope of 'manyn' it is not defined:

328. (manyn b WER ah e f IN the c BY the d) ≡ ((many b WER a e f IN g BY h) & (g I c) & (h I d))

Our first step in defining (manyn b IN g BY h WER a e f) is to change 'manyn' into 'a' and duplicate the sentence but negate the main relation in the duplicate:

(a b WER a e f IN g BY h) & (a b IN g BY h ~ WER a e f)

The 'a' which replaced 'manyn' in the positive biconditional is eliminated as follows:

329. (j W k) ≡ ((k I b) & (k WER a e f IN g BY h))
We now reduce \((k \ WER \ a^b \ e \ f \ IN \ g \ BY \ h)\) to standard form and place its equivalents back into the biconditional in the normal manner:

\[
(k \ WER \ a^b \ e \ f \ IN \ g \ BY \ h) \equiv ((k \ WER \ e \ p \ IN \ g \ BY \ h) \& (p \ I \ f)) \quad \text{DF a}
\]

\[
(k \ WER \ e \ p \ IN \ g \ BY \ h) \equiv ((k \ WER \ p \ IN \ g \ BY \ h) \& (p \ J \ e)) \quad \text{PCI}
\]

Hence 329 is equivalent to:

\[
(j \ W \ k) \equiv ((k \ I \ b) \& (k \ WER \ p \ IN \ g \ BY \ h) \& (p \ I \ f) \& (p \ J \ e))
\]

The reduction of \((a \ b \ IN \ g \ BY \ h \sim \ WER \ a^b \ e \ f)\) is more challenging:

330. \((m \ W \ n) \equiv ((n \ I \ b) \& (n \sim \ WER \ a^b \ e \ f \ IN \ g \ BY \ h))\)

Recall that \((b \sim R \ a \ c)\) is equivalent to \((b \ R \ no \ c)\). Hence, the following equivalences:

\[
(n \sim \ WER \ a^b \ e \ f \ IN \ g \ BY \ h) \equiv (n \ WER \ no \ e \ f \ IN \ g \ BY \ h)
\]

\[
(n \ WER \ no \ e \ f \ IN \ g \ BY \ h) \equiv (((o \ I \ f) \& (o \ J \ e)) \to (n \sim \ WER \ o \ IN \ g \ BY \ h))
\]

Thus 330 is equivalent to:

\[
(m \ W \ n) \equiv ((n \ I \ b) \& ((o \ I \ f) \& (o \ J \ e)) \to (n \sim \ WER \ o \ IN \ g \ BY \ h)))
\]

Now, admittedly we don't like the fact that there is a conditional embedded within the biconditional consequent and it will be an enormous challenge coding for this type of thing but I really see no way around it. We now must state that the \(j\) group is larger than the \(m\) group:

\[
(j \ GR \ m) \&
\]

The final step is that we use the Axiom of Indicative Tense and infer that there is a man in the van by the bridge who wears a black shirt and there is another man in the van who does not wear a black shirt. Hence, the full definition of \(\text{many}^n\) in 165 is:

\[
(\text{many}^n \ b \ IN \ g \ BY \ h \ WER \ a^b \ e \ f) \equiv
((j \ W \ k) \equiv ((k \ WER \ p \ IN \ g \ BY \ h) \& (k \ I \ b) \& (p \ I \ f) \& (p \ J \ e))) \&
(m \ W \ n) \equiv ((n \ I \ b) \& ((o \ I \ f) \& (o \ J \ e)) \to (n \sim \ WER \ o \ IN \ g \ BY \ h))) \&
(j \ GR \ m) \&
(q \ WER \ r \ IN \ g \ BY \ h) \& (q.s \ I \ b) \& (r.t \ I \ f) \& (r.t \ J \ e))
\]

Let's do another example:

331. I sang ah pretty song during the final minute of the first hour of \(\text{many}^n\) days during June.
It's a strange sentence but it'll do. The most important feature of this sentence is that we're using 'a\text{h}' rather than 'a'. If 'a' modified 'song' then it would not come under the scope of 'many\text{n}'. What this would mean is that I kept singing the same song and I know what it is I'm just not telling my interlocutor its identity. The more natural expression would be "I sang the same pretty song etc etc." Because I do not mean to say that I kept singing the same song, I have to superscript 'a' with 'h'.

332. \((b=\text{pretty}) \& (c=\text{song}) \& (d=\text{final}) \& (e=\text{minute}) \& (f=\text{first}) \& (g=\text{hour}) \& (h=\text{day}) \& (j=\text{June}) \& (k=\text{definite})\)

333. \((i \text{ SNG } a\text{h } b\text{ c DUR } d\text{ e OF } f\text{ g OF } many\text{h } h\text{ DUR } j) \equiv ((i \text{ SNG } a\text{h } b\text{ c DUR } d\text{ k OF } f\text{ m OF } many\text{h } h\text{ DUR } j) \& (k I c) \& (m I g))\)

334. \(((i \text{ SNG } a\text{h } b\text{ c DUR } d\text{ k OF } f\text{ m OF } many\text{h } h\text{ DUR } j) \equiv ((i \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } many\text{h } h\text{ DUR } j) \& (k J d) \& (m J f))\)

Because 'a\text{h}' comes under the scope of 'many\text{h}' we now must define it and the adjective following it at same time as 'many\text{n}'. Our first step is to eliminate 'many\text{h}':

335. \((n W o) \equiv ((o I h) \& (i \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } o\text{ DUR } j))\)

336. \((p W q) \equiv ((q I h) \& (i \sim \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j))\)

Let's first reduce the positive biconditional:

\((i \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } o\text{ DUR } j) \equiv ((r I c) \& (r J b) \& (i \text{ SNG } r\text{ DUR } k\text{ OF } m\text{ OF } o\text{ DUR } j))\)

DF 'a\text{h}', PRI

And now the negative biconditional:

\((i \sim \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j) \equiv (i \text{ SNG } no\text{ b c DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j)\)

DF \sim 'a\text{h}'

\((i \text{ SNG } no\text{ b c DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j) \equiv (((s I c) \& (s J b)) \rightarrow (i \sim \text{ SNG } s\text{ DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j))\)

DF 'no'

Hence, after we add on \((n GR p)\) and perform the Axiom of the Indicative Tense the definition of 'many\text{n}' is:

\((i \text{ SNG } a\text{h } b\text{ c DUR } k\text{ OF } m\text{ OF } many\text{n } h\text{ DUR } j) \equiv ((n W o) \equiv ((o I h) \& (r I c) \& (r J b) \& (i \text{ SNG } r\text{ DUR } k\text{ OF } m\text{ OF } o\text{ DUR } j)) \& (p W q) \equiv ((q I h) \& (((s I c) \& (s J b)) \rightarrow (i \sim \text{ SNG } s\text{ DUR } k\text{ OF } m\text{ OF } q\text{ DUR } j)) ) \& (n GR p) \& (i \text{ SNG } t\text{ DUR } k\text{ OF } m\text{ OF } u\text{ DUR } j) \& (u I h) \& (t I c) \& (i \sim \text{ SNG } v\text{ DUR } k\text{ OF } m\text{ OF } w\text{ DUR } j) \& (w I h) \& (v I c))\)

DF 'many\text{n}'
7. Narrow Implication: A Replacement of Material Implication

7.1. Historical Background

According to Sextus Empiricus it was Philo the Logician who first articulated the material conditional:

Philo, for example, said that the conditional is true when it does not begin with a true proposition and finish with a false one, so that a conditional, according to him, is true in three ways and false in one way. For when it begins with a true one and finishes with a true one, it is true, as in “If it is day, it is light.” And when it begins with a false one and finishes with a false one, it is again true for example, “If the earth flies, the earth has wings.” In the same way, too, the conditional that begins with a false one and finishes with a true one is true, such as “If the earth flies, the earth is.” But it is false only when it begins with a true one and finishes with a false one, as does “If it is day, it is night.”

More than 2200 years later, this theory is still orthodox as Tarski in his Introduction to Logic and to the Methodology of the Deductive Sciences writes:

By asserting an implication one claims that the antecedent cannot be true when the consequent is false. An implication is thus true in each of the following three cases: (i) both the antecedent and the consequent are true, (ii) the antecedent is false and the consequent is true, (iii) both the antecedent and the consequent are false; and only in the fourth possible case, when the antecedent is true and the consequent is false, is the whole implication false.

Another way to put this is:

337. p and q form a conditional relationship iff it is not the case that the antecedent is true and the consequent is false, further, one of the following three criteria are met, one, both the antecedent and the consequent are true, two, the antecedent is false and the consequent is true, three, both the antecedent and the consequent are false.

Tarski then admits that this theory affirms as true sentences which would 'hardly be regarded as meaningful, much less true':

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In order to illustrate the foregoing remarks, let us consider the following four sentences: "New York is a large city; New York is a large city; New York is a small city; New York is a small city."
In everyday language, these sentences would hardly be regarded as meaningful, much less as true. From the point of view of mathematical logic, on the other hand, they are all meaningful, the third sentence being false, while the remaining three are true. Of course, we do not thereby suggest that sentences like these are particularly relevant from any viewpoint whatever, or that we want to apply them as premises in our arguments.

It is interesting that he admits that he would not use the aforementioned as premises in an argument. I would like to know why. So now we have two classes of true conditionals: those that can be used as premises for arguments and those that cannot. Tarski owes us an explanation as to what demarcates these bad true conditionals from the good true conditionals. I think the demarcation is quite easy: the aforementioned conditionals are simply false.

In every introductory logic text I've seen the material conditional has won the day, but the authors are well aware that there is something wrong with it. In Hausman et al's *Logic and Philosophy*, now in its 10th edition, we find: "people do not in real life offer serious arguments with entirely disconnected conditionals as in the preceding examples."[39] In *A Concise Introduction to Logic*, now in its 13th edition, Hurley admits: "[T]he definitions of the five logical operators conform reasonably well with ordinary linguistic usage. However, as the last part of this section shows, the match is less than perfect."[316]

### 7.2. Narrow Implication

In this logic we put forward a competitor to material implication which I call narrow implication since under material implication 75% of all conjunctions composed of two sentences form conditional relationships whereas with narrow implication less than 1% do. As a first shot at the definition we will argue:

p and q form a conditional relationship iff the following four criteria are met:

1. It is consistent that p is contingently true and q is contingently true.
2. It is contradictory that p is contingently true and q is contingently false.
3. It is consistent that p is contingently false and q is contingently true.
4. It is consistent that p is contingently false and q is contingently false.

Let's test this theory. It is intuitively true that 'If I'm in San Diego on October 30, 1:07am 2017, then I'm in California on October 30, 1:07am in 2017'. (We need to link the relationship to time because it is conceivable that in the distant future San Diego will be in Mexico). In order for that conditional to be true then the following four criteria must be met and here we abbreviate the date with t:

1. It is consistent that it is contingently true that I'm in San Diego at time t and it is contingently true that I'm in California at time t.
2. It is contradictory that it is contingently true that I'm in San Diego at time t and it is contingently false that I'm in California at time t.
3. It is consistent that it is contingently false that I'm in San Diego at time t and it is contingently true that I'm in California at time t.
4. It is consistent that it is contingently false that I'm in San Diego at time t and it is contingently false that I'm in California at time t.

All four criteria are met and hence 'if I'm in San Diego at time t, then I'm in California at time t' is a true conditional. However, it should be pointed out that the material implication also correctly predicts that the aforementioned conditional is true. Where narrow implication demonstrates that it is superior to the material implication is correctly falsifying false conditionals. In the following, time t refers to the same date as the previous example:

338. It is false that 'if I'm in New York at time t, then I'm in California at time t'.

We will explore the criteria in a different order:

3. It is consistent that it is contingently false that I'm in New York at time t and it is contingently true that I'm in California at time t.
4. It is consistent that it is contingently false that I'm in New York at time t and it is contingently false that I'm in California at time t.

However, the aforementioned conditional does not meet criteria 1 and 3 of narrow implication:

1. It is false that it is consistent that it is contingently true that I'm in New York at time t and it is contingently true that I'm in California at time t.
2. It is false that it is contradictory that it is contingently true that I'm in New York at time t and it is contingently false that I'm in California at time t.

On the material account, 1a is true or false depending on who you are. If I'm neither in New York, nor in California, but in London, then 338 is a true conditional since it is false that I'm in New York and it is false that I'm in California. If I'm in California, again it is a true conditional since the consequent is true and the antecedent is false. But if I'm in New York, on the other hand, then it is a false conditional, since the antecedent will be true and the consequent will be false. This is certainly the wrong result.

Let's now do an example using legal reasoning:

339. It is false that if I'm a 16 year old US citizen, then I can vote in the US.
3. It is consistent that it is contingently false that I'm a 16 year old US citizen and it is contingently true that I can vote, e.g. I'm 21.
4. It is consistent it is contingently false that I'm a 16 year old US citizen and it is contingently false that I can vote, e.g. I'm 17.

However, once again, 4j does not meet criteria 1 and 3 for being a true conditional:
1. It is false that it is consistent that it is contingently true that I'm a 16 year old US citizen and it is contingently true that I can vote.

2. It is false that it is contradictory that it is contingently true that I'm a 16 year old US citizen and it is contingently false that I can vote.

The material implication once again makes the wrong prediction. If I'm 15 years old, then it is a true a conditional since it is false that I'm 16 and it is false that I can vote, but clearly if someone is 15 and a US citizen then it does not follow that they can vote.

Perhaps the problem with the material conditional is just the fact that it does not work when we use indexicals, since both of our examples so far have used them. I will now show that the problem with the material conditional is much greater than simply unable to handle conditionals which use indexicals:

340. It is false that if there are dogs, then there are cats.

Both the antecedent and the consequent are true but clearly one does not follow from the other.

1. It is consistent that it is contingently true that there are dogs and it is contingently true that there are cats.

3. It is consistent that it is contingently true that there are dogs and it is contingently false that there are cats.

4. It is consistent that it is contingently false that there are dogs and it is contingently true that there are cats.

However, 5j does not meet criteria 2 for being a true conditional:

2. It is false that it is a contradictory that it is contingently true that there are dogs and it is contingently false that there are cats.

The material implication incorrectly verifies 3a as a true conditional since both the antecedent and the consequent are true.

Let's now do an example using mathematical reasoning.

341. It is false that if \( x \) is greater than 4, then \( x \) is greater than 6.

341 should be false no matter what \( x \) is. Narrow implication verifies this intuition, while material implication does not.

1. It is consistent that it is contingently true that \( x \) is greater than 4 and it is contingently true that \( x \) is greater than 6, e.g., \( x = 7 \).

4. It is consistent that it is contingently false that \( x \) is greater than 4 and it is contingently false that \( x \) is greater than 6, e.g., \( x = 3 \).

But 341 does not meet the second and third criteria for being a true conditional:
2. It is false that it is contradictory that x is greater than 4 and it is contingently false that x is greater than 6, e.g., x = 5.
3. It is false that it is consistent that x is greater than 4 and it is contingently true that x is greater than 6.

Again, material implication sometimes verifies and sometimes does not. If x = 2, then 4a is true since it is false that 2 is greater than 4 and it is false that 2 is greater 6. On the other hand, if x = 5 then it is not a true conditional since the antecedent would be true and the consequent would be false. It is quite clear that the material conditional cannot verify mathematical statements where the value of the variable is unknown, a sure shortcoming.

Let's now see if narrow implication can correctly predict that two unrelated, false statements do not form a true conditional.

7j It is false that if the White House is in Oklahoma, then Big Ben is in Paris.

1. It is consistent that the White House is in Oklahoma and it is contingently true that Big Ben is in Paris, e.g., for example, using technology we do not have yet these buildings could be moved to new cities.
3. It is consistent that the White House is not in Oklahoma and it is contingently true that Big Ben is in Paris.
4. It is consistent that the White House is not in Oklahoma and it is contingently false that Big Ben is not in Paris.

However, 7j does not meet the second criterion for being a true conditional:

2. It is false that it is contradictory that the White House is not in Oklahoma and it is contingently false that Big Ben is in Paris.

Once again, material implication incorrectly verifies 7j since both the antecedent and the consequent are false, which makes 7j a true conditional according to material implication.

### 7.3. On the Simplicity of Narrow Implication

One of the reasons why Tarski likes material implication is because it is simple. He writes: "today it appears certain that the theory of material implication will surpass all other theories in simplicity; and, in any case, it must not be forgotten that logic, which has been founded upon this simple concept, turned out to be a satisfactory basis for the most complicated and most subtle of mathematical reasonings." [28] If we measure simplicity in terms of the number of words needed to explain the theory and whether or not those words are already known to everyone, even non-logicians, then both narrow implication and material implication are equal in simplicity. Narrow implication, however, does not lead to counterintuitive consequences and, as we will soon see, it solves eight paradoxes which material implication cannot solve. For the remainder of
this book I will assume that material implication has been successfully refuted.

### 7.4. Addressing the Circularities in Our Definition

There are problems with our definition. One, we do not know what it means to 'form a conditional relation' and we do not know what 'criteria' means. In order to avoid the fallacy of explaining the mysterious in terms of the mysterious we need to demystify 'criteria' and 'form a conditional relationship'. To say that \( p \) and \( q \) form a conditional relationship is to say \( p \rightarrow q \). In other words, I can translate the string of statement logic symbols \( (p \rightarrow q) \& q \) into '\( p \) and \( q \) form a conditional relationship and \( q \) is actual'. To understand 'criteria', suppose that sentence \( p \) is 'being a mammal'. We would then state the necessary and sufficient conditions for being a conditional on the right side of the \( \equiv \) sign like so: \( p \equiv (q \& r \& s \& t) \). The sentences, \( q \), \( r \) and \( s \) are criteria for being a mammal.

The next problem we face when trying to define the conditional is that definitions are formulated in the following form: \( p \equiv (q \& r) \) where \( q \) and \( r \) are more basic than \( p \). If a definition takes the following form \( (p \equiv q) \), then we say that it is a synonymous definition. So if a definition is formed using the \( \equiv \rightarrow \lor \land \# \) signs then what do we do when we try to define the \( \equiv \rightarrow \lor \land \# \) signs themselves?

What we do is we define \( \equiv \rightarrow \lor \land \# \) in terms of other symbols that we already have which are \& \& \& \& 'consistent'. Plus, we introduce one more symbol which is: \( \iff \). \( \iff \) allows us to do is make arguments where we're just worried about the relationship of the logical connectives themselves. We rarely make claims about the logical connectives, consequently, they need not be defined. But when we do make assertions about them, such as trying to argue for modus ponens we have to replace the logical connective with their definitions since \( \equiv \) equates the definiendum with its definiens, the \( \iff \) equates the definiendum of the logical connective with their definiens. The \( \iff \) simply allows us to replace the definiendum with the definiens and vice versa.

For the following definitions, these constants will be used:

\[
\begin{align*}
(r \iff p \& q) \& (s \iff p \& \neg q) \& (t \iff \neg p \& q) \& (u \iff \neg p \& \neg q) \& \text{(c=consistent)} \& \text{(d=contingent)}
\end{align*}
\]

\[
\begin{align*}
(p \# q) \iff ((r \& c) \& (s \& c) \& (t \& c) \& (u \& c) \& (p \& q \& d)) \\
(p \rightarrow q) \iff ((r \& c) \& (s \& \neg c) \& (t \& c) \& (u \& c) \& (p \& q \& d)) \\
(p \lor q) \iff ((r \& c) \& (s \& c) \& (t \& c) \& (u \& \neg c) \& (p \& q \& d)) \\
(p \equiv q) \iff ((r \& c) \& (s \& c) \& (t \& \neg c) \& (u \& c) \& (p \& q \& d)) \\
(p \lor q) \iff ((r \& \neg c) \& (s \& c) \& (t \& c) \& (u \& \neg c) \& (p \& q \& d))
\end{align*}
\]

We will pronounce one of them:

\( p \rightarrow q \) can be replaced with the following:

(a) \( p \) and \( q \) are consistent
(b) \( p \) and not \( q \) are not consistent
(c) not p and q are consistent  
(d) not p and not q are consistent  
(e) p is contingent  
(f) q is contingent

Recall that \((p \land q) \land (d=\text{contingent})\) is equivalent to 'p is contingent and q is contingent'. It is not equivalent to 'p and q are contingent at the same time' which does not make any sense, because 'being contingent' is always true.

### 7.5. The Definition of the # Sign, 'Unrelated'

For a long time I confused 'p does not follow from q' with 'p and q are unrelated'. While it is true that if 'p and q are unrelated', then 'p does not follow from q', but the converse is not true, which is to say, 'p does not follow from q' does not entail that 'p and q are unrelated' since it could be the case that q follows from p. We define 'unrelated' as follows:

p and q are unrelated iff the following four criteria are met:

1. It is consistent that p is contingently true and q is contingently true.  
2. It is consistent that p is contingently false and q is contingently true.  
3. It is consistent that p is contingently true and q is contingently false.  
4. It is consistent that p is contingently false and q is contingently false.

Let's test this theory.

5. 'I have a bike' and 'I'm in California' are unrelated.

6. It is consistent that 'I have a bike' is contingently true and 'I'm in California' is contingently true.  
7. It is consistent that 'I have a bike' is contingently false and 'I'm in California' is contingently true.  
8. It is consistent that 'I have a bike' is contingently true and 'I'm in California' is contingently false.  
9. It is consistent that 'I have a bike' is contingently false and 'I'm in California' is contingently false.

All four criteria for being unrelated are met and hence 5 constitutes a true conjunction of unrelated sentences. As usual, the material implication predicts that 5 is a true conditional provided both sentences are true and if 'I' should refer to the author on January 1, 2017, then it would be a true conditional, but if it should refer to someone who has a bike and is not in California then it is a not true conditional given material implication.
7.6. Contingency and Implication

Recall that p is contingent iff p is consistent and not p is consistent. Note that in our definition the negation of p has to be consistent. The negation of necessary statements must be contradictory. Hence, if the antecedent is necessary, then our definition cannot verify this. For example, suppose we have the conditional:

342. If 3 is the successor of 2 and if every number which is the successor of n is greater than n then 3 is greater than 2.

Now suppose we attempt to apply the second criterion to 1 to see if it is a true conditional, that is to say, negate the antecedent and affirm the consequent:

343. It is consistent that it is not the case that 3 is the successor of 2 and every number which is the successor of n is greater than n and 3 is greater than 2.

343 is false since it is not consistent that it is not the case that 3 is the successor of 2. The reason why we cannot use narrow implication to explain 1 is because we cannot consistently negate necessary statements and '3 is the successor of 2' is one such necessary statement. To handle this problem we simply need to recognize that when we say 'if p then q' where p is necessary then we need to use a new definition of the word 'if ... then'. For now, our definition is as follows:

\[(b \text{ EN } c) \equiv ((b \text{ W } d) \& (d \text{ J } e) \& ((b \text{ W } f) \rightarrow (f \text{ J } g)) \& (b \text{ c } J h) \& (b \text{ } \neg c \text{ } \sim J h)) \& (g=\text{true}) \& (h=\text{consistent}) \& (e=\text{necessary})\]

E.g. A set of premises b entails c iff all of the sentences in b are true and at least one of the sentences in b is necessary and b and c are consistent and b and not c are contradictory.

7.7. The Three Paradoxes of Strict Implication

Whereas narrow implication has four criteria, strict implication, discovered by C.I. Lewis in his landmark *Survey of Symbolic Logic*, only has one: "The strict implication, \(p \Rightarrow q\), means 'it is impossible that \(p\) be true and \(q\) be false,'

\[\text{or 'p is inconsistent with the denial of q',}[332]\] which is equivalent to the third criterion for being a narrow implication: 'it is contradictory that \(p\) is true and \(q\) is false.' Strict implication is certainly an improvement to material implication but it suffers from the following three counterexamples:

344. If there are dogs and there are no dogs, then \(2+2=5\).

which according to strict implication is equivalent to:
345. It is impossible that 'there are dogs and there are no dogs' is true and '2+2=5' is false.

345 is not a counterexample to strict implication since 'there are dogs and there are no dogs' is not true but the second formulation of strict implication is a counterexample:

346. 'there are dogs and there are no dogs' is inconsistent with the denial of '2+2=5'.

The second counterexample is:

347. If 2+2=4 then it is not the case that there are dogs and there are no dogs.

which according to strict implication is equivalent to:

348. It is impossible that it is true that '2+2=4' and it is false that 'it is not the case that there are dogs and there are no dogs'.

348 is not a counterexample to strict implication since it is true that it is not the case that there are dogs and there are no dogs. But the second formulation of strict implication is a counterexample:

349. '2+2=4' is inconsistent with the denial of 'it is not the case that there are dogs and there are no dogs'.

The third counterexample is falsifies strict implication given the first formulation:

350. If there are dogs and there are no dogs then there are cats and there are no cats.

which according to strict implication is equivalent to:

351. It is impossible that 'there are dogs and there are no dogs' is true and 'there are cats and there are no cats' is false.

Once again, 350 is false but 351 is true. In short, strict implication suffers from the following three paradoxes:

352. Any false statement follows from any contradiction.
353. The denial of any contradiction follows from any true statement.
354. Any contradiction follows from any other contradiction.

Tarski himself mentions strict implication but he does not take time to tell us what is wrong with it.
7.8. The Paradoxes of Material Implication: The First and Second Paradox

If p is true then it is entailed by any true proposition. \( p \rightarrow (q \rightarrow p) \)
If p is false then it entails any true proposition. \( \neg p \rightarrow (\neg p \rightarrow q) \)

Even a logician as capable as Russell believed this paradox. He writes: "Now if two terms are related in a certain way, it follows that, if they were not so related, every imaginable consequence would ensue. For, if they are so related, the hypothesis that they are not so related is false, and from a false hypothesis anything can be deduced."

We do not believe just any statement entails any other. We only believe this when we know what p and q imply. So each argument begins with premises and it just so happens that within our premises there are hypothetical statements. But these hypothetical statements do not simply connect any two statements whatsoever, rather they only connect those statements that meet the four criteria outlined above. We accept these connections without justification since in order to begin the process of justification we must start with some unjustified entailments.

So let's say our premises are \((a \rightarrow c) \& (c \rightarrow b)\). We now want to determine if a entails b. Since we know what a and b imply, then we need not simply accept on faith that a entails b because any true proposition entails any other, rather we can actually test to see if these two statements do share the relation of entailment. The first test is to affirm a and b and see if this results in consistency:

1. a
2. b
3. \( a \rightarrow c \)
4. \( c \rightarrow b \)

\[ \hline \]

5. c \hspace{1cm} \text{MP 1,3}
6. \( a \& b \& c \) \hspace{1cm} \text{&I 1,2,5}

Since \((a \& b \& c)\) is not a contradiction we have passed the first test. Our second test is to affirm a and deny b and see if this results in contradiction:

1. a
2. \( \neg b \)
3. \( a \rightarrow c \)
4. \( c \rightarrow b \)

\[ \hline \]

5. c \hspace{1cm} \text{MP 1,3}
6. \( \neg c \) \hspace{1cm} \text{MT 2,4}
7. \( \neg c \& c \) \hspace{1cm} \text{&I 5,6}
8. \( \bot \) \hspace{1cm} \text{\bot I 7}
We have passed the second test. Our third test is to deny a and affirm b and see if this results in consistency:

1. \(\neg a\)
2. \(b\)
3. \(a \rightarrow c\)
4. \(c \rightarrow b\)

\[\neg a \vee c \rightarrow E 3\]

\[\neg c \vee b \rightarrow E 4\]

\[\neg a \& b \& I\]

\(\neg a \vee c\) \& \(\neg c \vee b\) \& \(\neg a \& b\) is not a contradiction so we have passed the third test. Our fourth test is to deny a and deny b and see if this results in consistency:

1. \(\neg a\)
2. \(\neg b\)
3. \(a \rightarrow c\)
4. \(c \rightarrow b\)

\[\neg c \rightarrow E 2,4\]

\[\neg a \& \neg b \& \neg c \& I\]

\(\neg a \& \neg b \& \neg c\) is not a contradiction and hence is consistent. Hence, we have argued that a entails b. This demonstrates that not all true statements and not all false statements entail any other true statement, rather only statements with certain properties.

7.9. The Third Paradox

If p, q and r are three arbitrary propositions then either p implies q or q implies r.

\[(p \rightarrow q) \vee (q \rightarrow r)\]

We solve the third paradox by noticing that it contains an assumption which was falsified in our discussion of the first and second paradox. p does not imply q until we find out what sentences p and q are related to.

7.10. The Fourth Paradox

Since a disjunction is true when at least one of its disjuncts is true, then if p is true, then p and the addition of any disjunction is also true. p \(\rightarrow\) (p \(\vee\) q)

The false premise in the fifth paradox is: 'a disjunction is true when at least one of its disjuncts is true'. This is false. The definition for a disjunctive relationship is highly
similar to the definition of a conditional relationship. p and q share a disjunctive relation iff the following four criteria are met:

1. It is consistent that p is contingently true and q is contingently true.
2. It is consistent that p is contingently true and q is contingently false.
3. It is consistent that p is contingently false and q is contingently true.
4. It is contradictory that p is contingently false and q is contingently false.

Once again, since we do not know what sentences p and q are related to, then we are not going to get a contradiction when we negate both p and q, since (~p & ~q) is not a contradiction.

7.11. Ex Contradictione Nihil Sequitur

We do not have sufficient space here to explore many of the consequences of narrow implication but one consequence that must be addressed is whether or not from contradictions anything follows, or ex contradictione sequitur quod libet. Intuitively, it is false that anything follows from a contradiction. For instance, what does a rational non-logician do when they discover a contradiction in their beliefs? They alter their belief system so as to eliminate the contradiction or if they cannot figure out whether to give up p or not p, then they admit that they are agnostic regarding the truth of p. What they certainly do not do is start making conclusions. Not once in my life have I ever heard a non-logician assert that they know p because q and ~q is a contradiction. With narrow implication we do not have to adopt the absurdity that from contradictions anything follows, which we will explain in our solution to the fifth paradox. For the moment we will provide here an informal argument that nothing follows from contradictions:

1. If q is a valid inference from p, then if the premises of p are true, then q must be true.
2. Premises which contain a contradiction are never true.
3. Hence, no valid inference follows from premises which contain a contradiction.

We must distinguish, however, making inferences from contradictions which is invalid, as opposed to making inferences from contingently false statements. It is perfectly legitimate to wonder what the consequences would be if a certain contingently false statement were true. In fact, some of our most exciting science has been accomplished doing exactly that. Because Einstein wondered what would happen if he started traveling close to the speed of light, he was able to discover his relativity principle.

What logicians have done is they have failed to distinguish between contingently false statements and necessarily false statements. Only the former have consequences since they are sometimes true. Since the latter are never true, they have no consequences.

7.12. The Fifth Paradox

Anything follows from a contradiction. (p & ~p) → q
The short answer to solving this paradox is that our premises must not contain contradictions and \((p \& \sim p)\) is a contradiction. Although it is highly complicated, our senses tell us what contingent statements to believe. We then use logic to determine if our beliefs are consistent. Hence, all premises must contain only statements which we actually believe. If we want to do a thought experiment to determine what would be the consequences of a certain counterfactual, then we nevertheless believe that these statements are consistent, just not actual.

The long answer involves determining whether or not narrow implication can falsify this paradox. So let's say the contradiction is: 'there are dogs and there are no dogs' and the consequent is: 'Marilyn Monroe is the most beautiful woman'. If the second follows from the contradiction then the following four criteria should be met:

2. It is true that it is consistent that it is not the case that there are dogs and there are no dogs and Marilyn Monroe is the most beautiful woman.
3. It is true that it is contradictory that there are dogs and there are no dogs and Marilyn Monroe is not the most beautiful woman.
4. It is true that it is consistent that it is not the case that there are dogs and there are no dogs and Marilyn Monroe is not the most beautiful woman.

However, the fifth paradox does not meet the first criterion for being a true conditional:

1. It is false that it is consistent that there are dogs and there are no dogs and Marilyn Monroe is the most beautiful woman.

There are two culprits for the origin of this erroneous belief. One, logicians confuse 'not' with the word 'contradictory', or sometimes false with always false. Something can be false but still consistent, or all contradictions are false, but not all falsehoods are contradictions. For example, 'There are seven planets' is false but not a logical contradiction. We have dealt with this problem in our logic by using \(\bot\) to stand for contradiction and \(\sim\) to stand for negation. The second culprit is the following deduction is believed by some to be valid:

5. \(p \& \sim p\)
6. \(p\) \&E 5
7. \(p \lor q\) \lor I 6
8. \(\sim p\) \lor E 6,7

The origin of this deduction comes from Thomas of Erfurt, also known as Pseudoscot, in his *De Modis Significandi* published in 1310. However, I have that on the authority of Kneale and Kneale since I could not find the following words in my copy of the text and the following quote comes from the collected works of Duns Scotus of which Erfurt's text is believed to have been spuriously included:

Probatur quia sequitur Socrates est et Socrates non est, igitur Socrates non est, quia a copulativa ad alteram eius partem est consequentia formalis. Tunc
There are two things wrong with 5-8. First, 5 can never be a premise since whenever there are contradictions within our belief system, we must give up one of them, that is to say, that the proper thing a logician should do when they discover a contradiction within their belief system is to go back to the drawing board, spot the source of the contradiction and abandon the belief that causes it. Second, 7 is an invalid inference as we explained above since one of the necessary conditions for p and q forming a true disjunction is that (~p & ~q) must be a contradiction and they are not. As Daniel Nagase pointed out to me, the moral of the story is to simply remove contradictions from one's premises which everyone outside of the logical community does anyway.

What we have is a classic example of logicians abandoning the belief that they are more certain of in favor of a belief they are less certain of. This would be like a scientist who discovers an equation to explain the measurements of data. They are certain of the measurements, not very certain of their equation. So when their equation predicts some of the measurements but then stops working they refuse to deny their equation and instead assert that the measurements are wrong. Similarly, we are trying to explain something which is very difficult to explain: what do all sentences of the form 'if p then q' have in common? There are already a variety of sentences which use this word and of which we are dead certain that they are incorrect, one of these is: "If 2+2=5, then New York is a large city." When we formulate our hypothesis and we cannot get it to falsify that sentence, then we abandon the hypothesis. We do not hold on to the hypothesis and assert that 'If 2+2=5, then New York is a large city' is in fact correct.

7.13. The Sixth Paradox

Since any true belief follows from any false belief, it follows that ~p → p

Even more shocking is that some logicians actually believe that 'there are dogs' follows from 'there are no dogs'. We solve this problem in the following manner. In order for p to be a logical consequence of ~p then it must meet the four criteria for being a true
entailment: first, if I affirm the antecedent and affirm the consequent, then I must get consistency but instead I get contradiction, like so:

1. $\neg p & p \bot$

We have already shown that $\neg p \rightarrow p$ does not meet all of the four criteria for being a sound conditional. Nevertheless, let's see what happens when we apply the other tests. Second, if I deny the antecedent and affirm the consequent I must get consistency which I do:

2. $\neg\neg p & p \mathcal{C}$

So $\neg p \rightarrow p$ passes the second test. Third, if I affirm the antecedent and deny the consequent then I must obtain contradiction but instead I obtain consistency:

3. $\neg p & \neg p \mathcal{C}$

Fourth, if I deny the antecedent and deny the consequent, then I must get consistency but once again I obtain contradiction:

4. $\neg\neg p & \neg p \bot$

All in all $\neg p \rightarrow p$ fails 3 of the 4 tests needed to be a true entailment.

7.14. The Seventh Paradox

Either $q$ or its negation is true, so their disjunction is implied by any true proposition.

$p \rightarrow (q \lor \neg q)$

In this system $(q \lor \neg q)$ is contradictory and $(q \nand \neg q)$ is consistent. So let's ask if we can solve the eight paradox in the following form:

$p \rightarrow (q \nand \neg q)$

Because we know nothing about $p$'s relations then it follows that we cannot make any inferences with $p$ as the sole premise.
Works Cited


